



Geel 2000 Language Schools

Math Department

Second Term

Sec. 1



2025/2026

Name :-----

Class:-----

Date ://



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ALGEBRA



Lesson (1) : Matrices

The matrix: "Is an organization of some elements written in rows and columns between brackets in the form ()".

Ex:

1 st column	2 nd	3 rd	
-5	3	10	→ 1 st row
1	4	-4	→ 2 nd row
0	$\sqrt{3}$	7	→ 3 rd row

The order of any matrix = no. of rows x no. of columns

How to express the elements in the matrix.

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$

a_{32}
 ↙ ↘
 row column

∴ a_{32} is the element in 3rd row and the 2nd column.

Some types of matrices:

- A Square matrix:** It is a matrix in which the number of its rows equals the number of its columns. For example: $\begin{pmatrix} -3 & 2 \\ 4 & -1 \end{pmatrix}$ (a 2×2 square matrix)
- B Row matrix:** It is a matrix containing one row and any number of columns. For example : (2 4 6 8) (a 1×4 row matrix)
- C Column matrix:** It is a matrix containing one column and any number of rows. For example: $\begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}$ (a 3×1 column matrix)
- D Zero matrix:** It is a matrix in which all of its elements are Zeros. It may be a square matrix or not. For examples:
(0) is a 1×1 zero matrix, (0 0) is a 1×2 zero matrix, $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is a 2×1 zero matrix, $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is a 2×2 square zero matrix and is denoted by $\mathbf{O}$.
- E Diagonal matrix:** It is a square matrix in which all elements are zeros except the elements of its diagonal then at least one of them is not equal to zero. For example:
the matrix: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ (is a 3×3 diagonal matrix)
- F Unit matrix:** it is a diagonal matrix in which each element on the main diagonal has the numeral 1, while 0 exists in all other elements , it is denoted by I. for example:
each of:
(1) , $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is a unit matrix.

Transpose of matrix:

If $A = (a_{xy})$ then $A^T = a_{yx}$

Where A^T is the transpose of A

Note: $(A^T)^T = A$

Ex1: Write the matrix (A_{xy}) of the dimensions 3×2 where: $a_{xy} = 2x - y$

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Ex2: Write the matrix (B_{xy}) of the order 3×3 where: $b_{xy} = 3x - 2y$

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Ex3: Find the transpose of the following matrices and write its order:

$$A = \begin{pmatrix} 2 & 3 & 0 \\ -1 & 5 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 9 \\ -2 \\ 4 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} -7 & 5 \\ 9 & 4 \end{pmatrix}$$

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The equality of two matrices

If A and B are two matrices then $A = B$ if and only if

- 1- A and B with the same order
- 2- The corresponding elements are equal.

$$\begin{pmatrix} 1 & 0 & -2 \\ 2 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -2 \\ 2 & 3 & -1 \end{pmatrix}$$

Ex1: Find the values of x, y and Z if

$$\begin{pmatrix} 7 & 0 & 2 \\ 4 & 7 & 5 \end{pmatrix} = \begin{pmatrix} -1 & 0 & X+5 \\ 4 & 2y-3 & 5 \end{pmatrix}$$

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Ex2: If $X = \begin{pmatrix} 3a+1 & 12-b & h^3 \\ c+2d & 18 & 6 \end{pmatrix}$ $Y = \begin{pmatrix} 1 & 9 \\ 3 & 18 \\ -8 & d+2c \end{pmatrix}$

Find a , b , c , d and h if $X = Y^T$

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Symmetric and skew symmetric matrices:

If A is a square matrix , then

- A is called a symmetric matrix if and only if $A = A^T$
- A is called a skew symmetric matrix if and only if $A = -A^T$

$$A = \begin{pmatrix} 2 & -1 & -3 \\ -1 & 4 & 0 \\ -3 & 0 & 5 \end{pmatrix} \text{ is symmetric matrix} \quad B = \begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & \frac{1}{2} \\ 2 & -\frac{1}{2} & 0 \end{pmatrix} \text{ is skew}$$

symmetric

"Dot Product"

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 \end{bmatrix}$$

sheet (1)**Choose the correct answer from those given :**

(1) If $A = \begin{pmatrix} 1 & 1 & x-1 \\ 1 & 3 & 5 \\ -1 & 5 & 6 \end{pmatrix}$ is a symmetric matrix , then $x = \dots\dots\dots$

- (a) - 1 (b) zero (c) 4 (d) 6

(2) If $A = \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 \\ \frac{1}{2}k & 4 \end{pmatrix}$ where $A = B^t$, then $k = \dots\dots\dots$

- (a) - 2 (b)
- $-\frac{3}{2}$
- (c) 8 (d) - 6

(3) If $A = \begin{pmatrix} 1 & 5 \\ 3 & 2 \\ -1 & 7 \end{pmatrix}$, then $a_{12} + a_{32} = \dots\dots\dots$

- (a) 8 (b) 12 (c) zero (d) 10

(4) If $\begin{pmatrix} 1 & x & 2 \\ -1 & 6 & 5 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -3 & 6 \\ 2 & y \end{pmatrix}^t$, then $xy = \dots\dots\dots$

- (a) - 15 (b) - 2 (c) 2 (d) 15

Complete the following :
(1) If A is a matrix of order 2×2 and if $a_{11} = 3$, $a_{12} = 5$, $a_{21} = \frac{1}{2}$ and $a_{22} = \sqrt{5}$, then the matrix A =

(2) If A is a matrix of order 3×2 and if $a_{11} = 2$, $a_{21} = 3$, $a_{32} = \frac{1}{2} a_{11}$, $a_{22} = a_{21} + 3$, $a_{31} = -9$, $a_{12} = \frac{1}{3} a_{31}$, then the matrix A =

(3) If $Y = \begin{pmatrix} -4 & 2 & 5 \\ 1 & -\sqrt{3} & 9 \end{pmatrix}$, then the matrix Y is of order
 $y_{21} = \dots\dots\dots$, $y_{22} = \dots\dots\dots$, $\frac{1}{2} y_{12} + \sqrt{y_{23}} = \dots\dots\dots$

(4) If A is a matrix of order 2×3 , then the number of elements of the matrix A is

(5) If B is a matrix of order 3×1 , then B^t is a matrix of order

(6) If O is a zero matrix of order 3×3 , then $O^t = \dots\dots\dots$

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3] If $A = \begin{pmatrix} 5 & 2x & 8 \\ -4 & -3 & 6 \\ x+2y & 6 & 4 \end{pmatrix}$ is a symmetric matrix , then Find the value of : x , y

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4] If $B = \begin{pmatrix} 0 & 3x & 7 \\ x+3 & 0 & -2z \\ 3y-x & 6 & 0 \end{pmatrix}$ is skew symmetric matrix Find the value of x , y

and z

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Lesson (2) : Operation on matrices

I-Addition and subtraction:

To add two matrices A, B they must have the same order.

Ex1: If $A = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 6 & -7 \\ 4 & 3 \end{pmatrix}$

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Ex2: If $A = \begin{pmatrix} 2 & -2 \\ 4 & 6 \end{pmatrix}$, Find $3A$

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Ex3: If $A = \begin{pmatrix} 3 & 1 \\ 2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 3 \\ 4 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & 5 \\ -2 & 4 \end{pmatrix}$

Find: (1) $A + B$ (2) $B - C$ (3) $A + 2B - C$

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Sheet (2)**I-Complete:**

1) If $A + \begin{pmatrix} -3 & -2 \\ 5 & 4 \end{pmatrix} = 0$, then $A = \dots\dots\dots$

2) If O is the Zero matrix of order 2×2 , then $4O = \dots\dots\dots$ and it is of order.....3) If each of the matrices A and B is of order 3×1 , then the resultant matrix of $A - 2B$ is of order.....

4) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}^T = \dots\dots\dots$ which is of order.....

5) If $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, then $3A = \dots\dots\dots$, $-2A = \dots\dots\dots$

6) If $A = \begin{pmatrix} 15 & 10 \\ 5 & 20 \end{pmatrix}$, then $A = 5 \begin{pmatrix} \dots\dots\dots & \dots\dots\dots \\ \dots\dots\dots & \dots\dots\dots \end{pmatrix}$

2] If $A = \begin{pmatrix} -3 & 1 \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$

Check that: 1) $(A + B)^T = A^T + B^T$ 2) $A - B \neq B - A$

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3] If $\begin{pmatrix} 3 & 6 \\ 5 & -7 \end{pmatrix} + \begin{pmatrix} 1 & -4 \\ -2 & 6 \end{pmatrix} = \begin{pmatrix} X & 4 \\ 7 & Y \end{pmatrix}$

Find the value of X and Y.

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4] Find X, Y, Z, and L that satisfy that:

$$X \begin{pmatrix} 1 & 3 \\ 5 & Y \end{pmatrix} + Z \begin{pmatrix} 2 & L \\ 0 & 4 \end{pmatrix} + 5 \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = O_{2 \times 2}$$

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7] If $A = \begin{pmatrix} 5 & -3 & 6 \\ 2 & 5 & 0 \\ 4 & -2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 1 & 3 \\ -4 & 21 & -5 \\ 3 & 12 & 6 \end{pmatrix}$

Find the matrix X such that: $3A + X = 2B$

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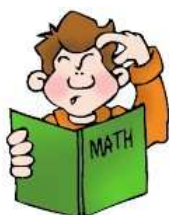
8] If $X + 2X^T = \begin{pmatrix} 9 & 14 \\ 13 & 6 \end{pmatrix}$, find the matrix X.

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9] If $A = \begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix}$ and $B^T = \begin{pmatrix} -1 & 0 \\ 3 & 4 \end{pmatrix}$,

Find the matrix X such that: $4X - 3B + 2A^T = A + (5B - X)^T$.

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Problem Solving/
Logical Thinking
at pppst.com

Lesson (3) : Multiplying Matrices

✚ If A is a matrix of order $m \times n$, B is a matrix of order $r \times L$, then their product $C = A \times B$ will be defined if and only if $n = r$

✚ To multiply two matrices A no. of columns = no. or rows B

 2×3 3×1

2x1 is the order of the product matrix

Ex1: If $A = \begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 1 \\ 5 & 6 \end{pmatrix}$

Ex2: If $A = \begin{pmatrix} 3 & -2 \\ 0 & 2 \\ -1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -1 \\ 5 & 7 \end{pmatrix}$, $C = \begin{pmatrix} 4 & 0 & 3 \\ 5 & 2 & -1 \end{pmatrix}$

and $D = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ Check that: 1) $(AB)^T = B^T A^T$ 2) $(AB)C =$

$A(BC)$

Sheet (3)**1 Complete the following :**

- (1) If A is a matrix of order $m \times n$ and B is a matrix of order $r \times l$
 , then AB is defined if and AB is undefined if
- (2) If A is a matrix of order 3×1 and B is a matrix of order 1×3
 , then AB is a matrix of order and BA is a matrix of order
- (3) If A is a matrix of order 2×3 and AB is defined as a matrix of order 2×1
 , then B is a matrix of order
- (4) If A is a matrix of order 2×3 and B^t is a matrix or order 1×3
 , then AB is a matrix of order
- (5) If A is a square matrix , I is the identity matrix of the same order of A ,
 then $A \times I = I \times A = \dots\dots\dots$, $I^t = \dots\dots\dots$, $I^2 = \dots\dots\dots$
 , $I^3 = \dots\dots\dots$, $I^n = \dots\dots\dots$ where n is a positive integer.

2] If $X = \begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix}$ and $Y = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$, **prove that :** $XY \neq YX$.

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3] If $X = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$ and $Y = \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}$, **find :** $X^2 - Y^2$

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Lesson (4) : Determinants

second order

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Ex:1 Find the value of the following determinant :

a) $\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}$

b) $\begin{vmatrix} 4 & -7 \\ 2 & 6 \end{vmatrix}$

c) $\begin{vmatrix} 5 & 4 \\ -3 & -2 \end{vmatrix}$

d) $\begin{vmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{vmatrix}$

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• Third order

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & j \end{vmatrix} = a \begin{vmatrix} e & f \\ h & j \end{vmatrix} - b \begin{vmatrix} d & f \\ g & j \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= -a \begin{vmatrix} e & f \\ h & j \end{vmatrix} + b \begin{vmatrix} d & f \\ g & j \end{vmatrix} - c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Ex:2 Find the value of the following determinant :

a) $\begin{vmatrix} 4 & -1 & 3 \\ 0 & 5 & -2 \\ 0 & -3 & -1 \end{vmatrix}$

b) $\begin{vmatrix} -1 & 2 & 5 \\ 0 & 3 & -2 \\ 0 & 0 & 6 \end{vmatrix}$

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□ Another method

$$\begin{vmatrix} a & b & c \\ d & e & l \\ m & n & k \end{vmatrix} \xrightarrow{\text{Repeat the first two}} \begin{vmatrix} a & b & c & a & b \\ d & e & l & d & e \\ m & n & k & m & n \end{vmatrix}$$

$$S1 = aek + blm + cdn$$

$$S2 = bdk + aln + cem$$

Then the value of the determinant is $S = S1 - S2$

➤ Remark :

(1) The triangular matrix:

It is a square matrix in which elements above or below principal diagonal are zeroes

$$\text{Ex) } \begin{pmatrix} a & 0 \\ c & d \end{pmatrix}, \begin{pmatrix} a & b & c \\ 0 & e & l \\ 0 & 0 & k \end{pmatrix}$$

$$\text{Its determinant} = \begin{vmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{vmatrix} = a_{11} \times a_{22}$$

$$\text{And } \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{vmatrix} = a_{11} \times a_{22} \times a_{33}$$

(2) Finding the area of triangle using determinants:

If ΔABC in which $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$

$$\text{Then the area of triangle } ABC = \frac{1}{2} |A| \text{ where } A = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Steps:

$$\text{a) Find } A = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\text{b) Area} = \frac{1}{2} |A|$$

Note: use elements of the 3rd column because it is easier

(3) To prove that three points are collinear:

The three points (x_1, y_1) , (x_2, y_2) and $C(x_3, y_3)$ are collinear if

$$A = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \text{zero}$$

➤ Cramer's rule**First: solving a system of linear equations of two variables:**

To solve the two equations $ax + by = m$ and $cx + dy = n$ follow the steps:

1) Find the three determinants Δ , Δx and Δy where

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}, \Delta x = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta y = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta \neq 0$$

2) To find the value of x , y

$$x = \frac{\Delta x}{\Delta}, y = \frac{\Delta y}{\Delta}$$

Note: If $\Delta = 0$ then the system has no solution

Second: solving a system of linear equations of three variables:

To solve the two equations $a_1x + b_1y + c_1z = m$, $a_2x + b_2y + c_2z = n$ and $a_3x + b_3y + c_3z = k$ follow the steps:

1) Find the four determinants Δ , Δx , Δy and Δz where

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta x = \begin{vmatrix} m & b_1 & c_1 \\ n & b_2 & c_2 \\ k & b_3 & c_3 \end{vmatrix}, \Delta y = \begin{vmatrix} a_1 & m & c_1 \\ a_2 & n & c_2 \\ a_3 & k & c_3 \end{vmatrix}$$

$$\Delta z = \begin{vmatrix} a_1 & b_1 & m \\ a_2 & b_2 & n \\ a_3 & b_3 & k \end{vmatrix}, \Delta \neq 0$$

2) To find the value of x , y and z

$$x = \frac{\Delta x}{\Delta}, y = \frac{\Delta y}{\Delta}, z = \frac{\Delta z}{\Delta}$$

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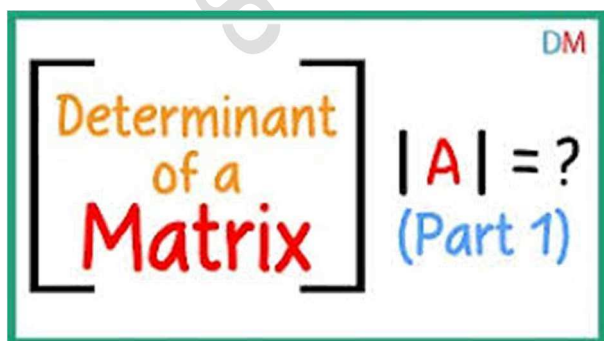
Ex:3 solve the equation :
$$\begin{vmatrix} x & 0 & 1 \\ 8 & 1-x & -x \\ x & -1 & 1+x \end{vmatrix} = 0$$

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Ex:4 Find the area of a triangle whose vertices are

X(1,2) ,Y(3,-4) and Z(-2,3)

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Sheet 4**1 Find the value of each of the following determinants :**

$$(1) \begin{vmatrix} 7 & 5 \\ 3 & 2 \end{vmatrix}$$

$$(2) \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$$

$$(3) \begin{vmatrix} -2 & -2 \\ 4 & 0 \end{vmatrix}$$

2 Prove that :

$$(1) \begin{vmatrix} 2x & -1 \\ 2 & 3x \end{vmatrix} + \begin{vmatrix} 3 & 6x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 13 \\ -2 & -7 \end{vmatrix}$$

$$(2) \begin{vmatrix} \csc \theta & \cot^2 \theta \\ 1 & \csc \theta \end{vmatrix} \times \begin{vmatrix} 2 & -3 \\ 5 & -7 \end{vmatrix} = 1$$

3 Find the value of each of the following determinants

$$(1) \begin{vmatrix} 1 & 2 & 3 \\ -1 & 4 & 4 \\ 0 & 7 & 8 \end{vmatrix}$$

$$(2) \begin{vmatrix} 0 & 42 & 3 \\ 2 & 18 & 7 \\ 0 & 28 & 3 \end{vmatrix}$$

4 Solve each of the following equations

(1) $\begin{vmatrix} 2 & 1 \\ 4 & x \end{vmatrix} = 0$

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(2) $\begin{vmatrix} x & -1 \\ 2 & x \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 2 & x \end{vmatrix} = 2$

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(3) $\begin{vmatrix} 0 & -1 & x \\ x & 4 & 3 \\ 2 & 1 & 2 \end{vmatrix} = 10$

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Find using determinants the area of the triangle :

(1) A (2 , 4) , B (- 2 , 4) , C (0 , - 2)

(2) X (3 , 3) , Y (- 4 , 2) , Z (1 , - 4)

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Use determinants to prove that each of the following points are collinear :

(1)  (3 , 5) , (4 , - 1) , (5 , - 7)

(2) (3 , 2) , (- 1 , 0) , (- 5 , - 2)

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Solve each of the following systems of linear equations by Cramer's rule :

(1) $2x - 3y = 5$, $3x + 4y = -1$

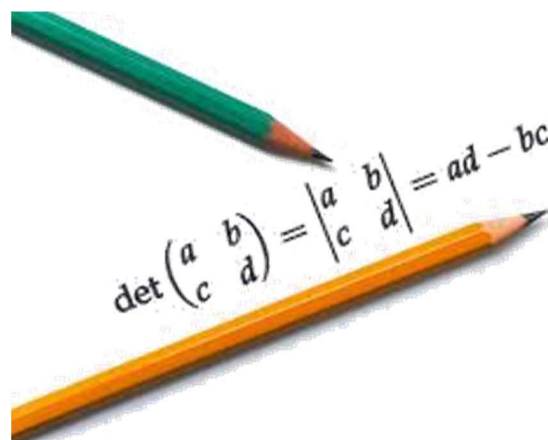
(2) $x + 3y = 5$, $2x + 5y = 8$

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Solve each of the following systems of linear equations by Cramer's rule :

(1) $2x + y - 2z = 10$, $3x + 2y + 2z = 1$, $5x + 4y + 3z = 4$

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Lesson (5) : Multiplicative inverse of a matrix

$$\text{If } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ Then } A^{-1} = \frac{1}{\Delta} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad \begin{matrix} AA^{-1} = A^{-1}A = I \\ \Delta \neq 0 \end{matrix}$$

1] Show the matrix which have multiplicative inverse :

a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix}$

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d) $\begin{pmatrix} 2 & 6 \\ -1 & 3 \end{pmatrix}$

e) $\begin{pmatrix} -1 & 0 \\ 3 & 4 \end{pmatrix}$

f) $\begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$

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2] what is the real values of a which make each of the following matrices has A multiplicative inverse :

a) $\begin{pmatrix} a & 1 \\ 6 & 3 \end{pmatrix}$

b) $\begin{pmatrix} a & 9 \\ 4 & a \end{pmatrix}$

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3] if : $X = \begin{pmatrix} 1 & x \\ 0 & -x \end{pmatrix}$ prove that : $X^{-1} = X$

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4] solve each of the following system using the matrices :

a) $3x+2y=5$, $2x+y=3$

b) $2x-7y=3$, $x-3y=2$

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Sheet 5

- 1** Show the matrices which have multiplicative inverse and the matrices which have not multiplicative inverse in the following , and find it if it is existed :

(1) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

(2) $\begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$

(3) $\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix}$

- 2] Find the real values of x which make the matrix $\begin{pmatrix} x & 27 \\ 3 & x \end{pmatrix}$ have no multiplicative inverse.

- 3] If $X = \begin{pmatrix} 1 & x \\ 0 & -1 \end{pmatrix}$, prove that : $X^{-1} = X$

4] If $A = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$ and $AB = \begin{pmatrix} 4 & -2 \\ 0 & 7 \end{pmatrix}$, **find the matrix B**

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5] **Solve each system of the following linear equations using the matrices :**

(1)  $3x + 2y = 5$, $2x + y = 3$ | (2)  $2x - 7y = 3$, $x - 3y = 2$

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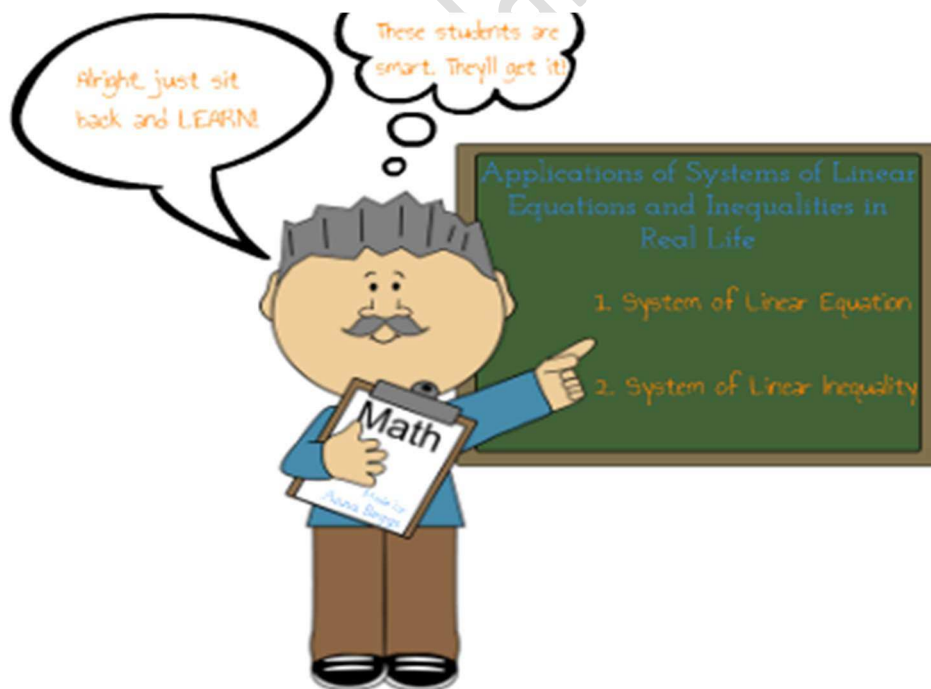
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LINEAR PROGRAMMING



Lesson (1): linear inequality

First: Inequality of first degree in one variable

Example

- ① Find the solution set of each of the following inequalities where $x \in \mathbb{R}$ then represent the solution on the number line:

A $3x - 9 > 6x$

B $6 + x < 3x + 2 \leq 14 + x$

Solution

A $3x - 9 > 6x$

add $(9 - 6x)$ to both sides.

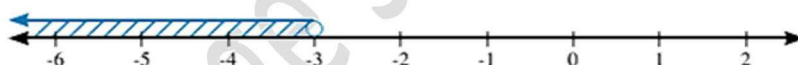
$$\therefore 3x - 9 + 9 - 6x > 6x + 9 - 6x$$

$$\therefore -3x > 9$$

(multiply both sides by $-\frac{1}{3}$)

$$x < -3$$

the solution set = $] -\infty, -3[$



- B Divide the inequality into two inequalities as follows:

The first inequality: $6 + x < 3x + 2$

The second inequality: $3x + 2 \leq 14 + x$

$$\therefore 6 - 2 < 3x - x$$

$$\therefore 3x - x \leq 14 - 2$$

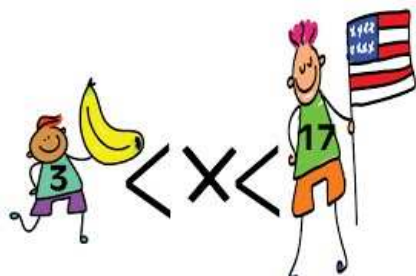
$$\therefore x > 2$$

$$\therefore x \leq 6$$

The solution set = $]2, \infty[$

The solution set = $] -\infty, 6]$

The solution set = $]2, \infty[\cap] -\infty, 6] =]2, 6]$



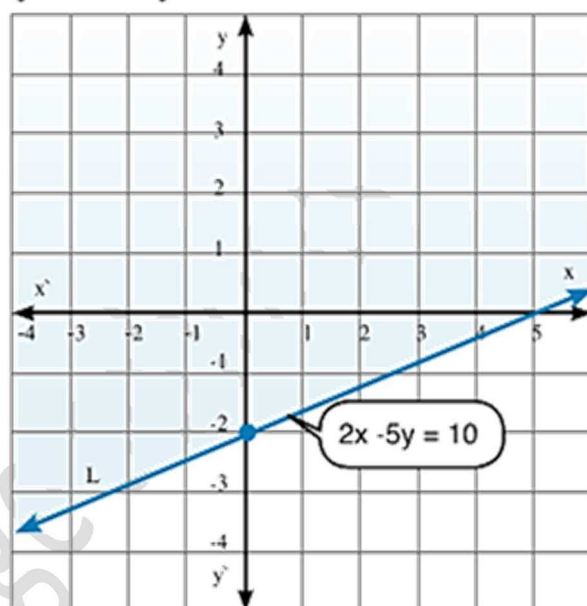
Second : Inequality of first degree in two variables**Example**) Represent graphically the solution set of the inequality: $2x - 5y \leq 10$ **Solution****Step (1):** represent graphically the boundary line (L). $2x - 5y = 10$ by a solid line (because the inequality relation $\leq$).

x	0	5	$2\frac{1}{2}$
y	-2	0	-1

You can draw the boundary line, write the straight line:

 $2x - 5y = 10$ in the form: $y = mx + c$

where m is the slop and c is the y - intercept from the y-axis.

then: $-5y = -2x + 10 \quad \therefore y = \frac{2}{5}x - 2$ **Step (2):** test the point (0,0) which lies on one side of the boundary line. $2x - 5y \leq 10$ (the original inequality) $2(0) - 5(0) \stackrel{?}{\leq} 10$ (substitute the point (0, 0)) $0 \leq 10$ (True)Colour the region which contains the point (0, 0), where the solution set is half the plane which the point (0, 0) lies $\cup$ the set of points on the boundary line L.

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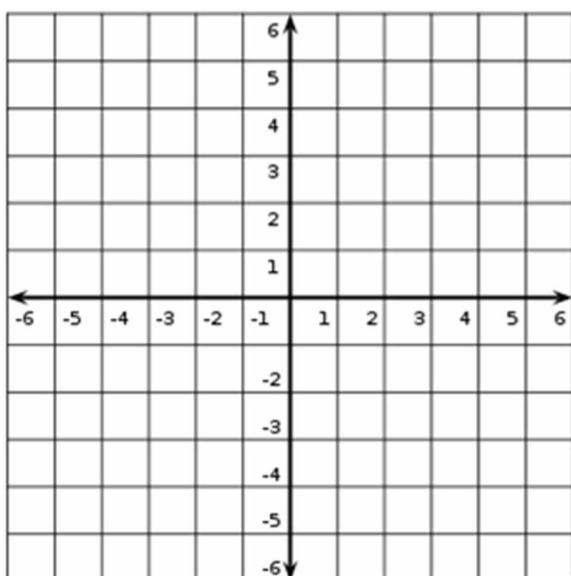


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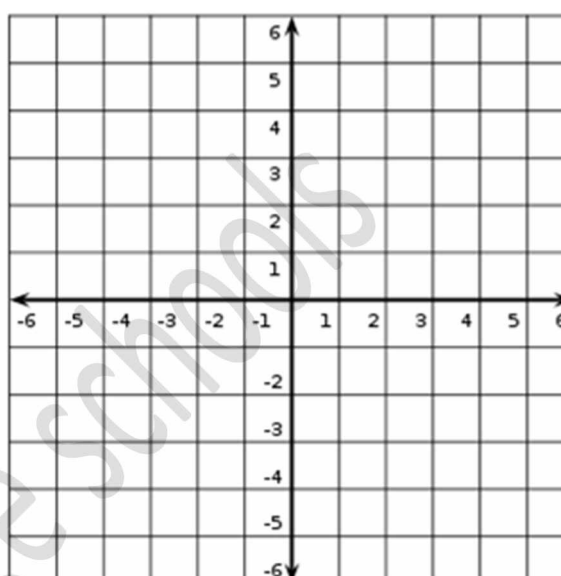
Sheet (1)

(1) Find graphically the S.S of each of the following :

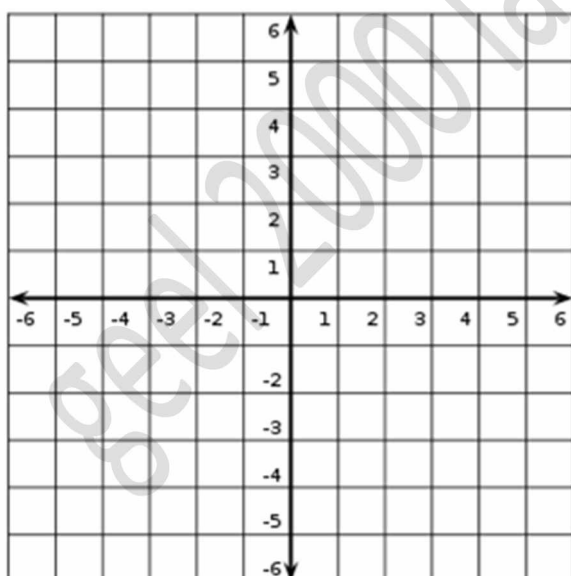
a) $x \geq -2$



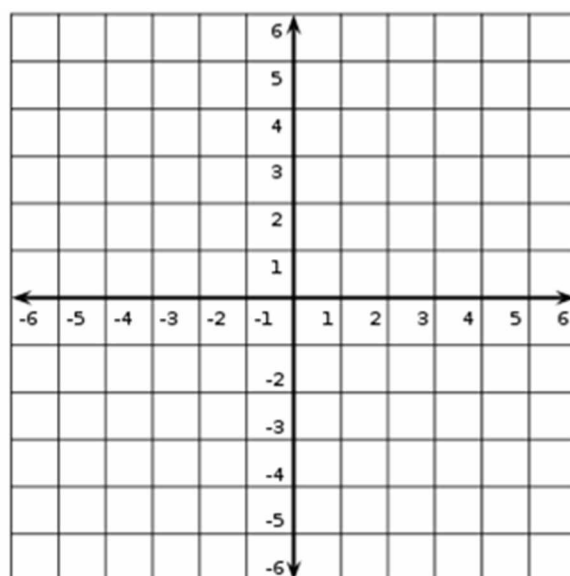
b) $y < 5$



c) $-1 < x \leq 3$



d) $0 \leq y \leq 4$



Solving system of linear inequalities graphically

To solve two or more linear inequalities graphically do the following steps:

- 1) Shade the region S_1 that represents the S.S of the 1st inequality.
- 2) Shade the region S_2 that represents the S.S of the 2nd inequality.
- 3) The common region S of the two regions S_1 and S_2 represents the S.S of the two inequalities where: $S = S_1 \cap S_2$ as the opposite figure:

Very important remarks:

- The eqn. $y = 0$ is represented by X- axis
- The eqn. $X = 0$ is represented by y- axis

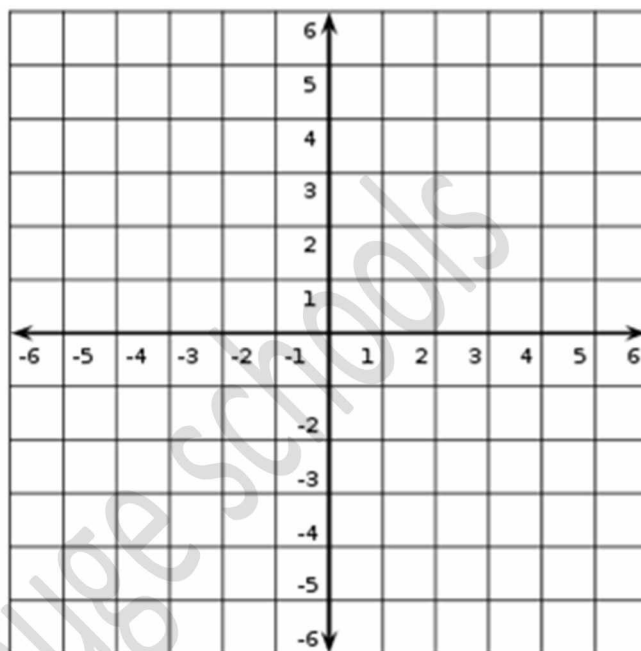
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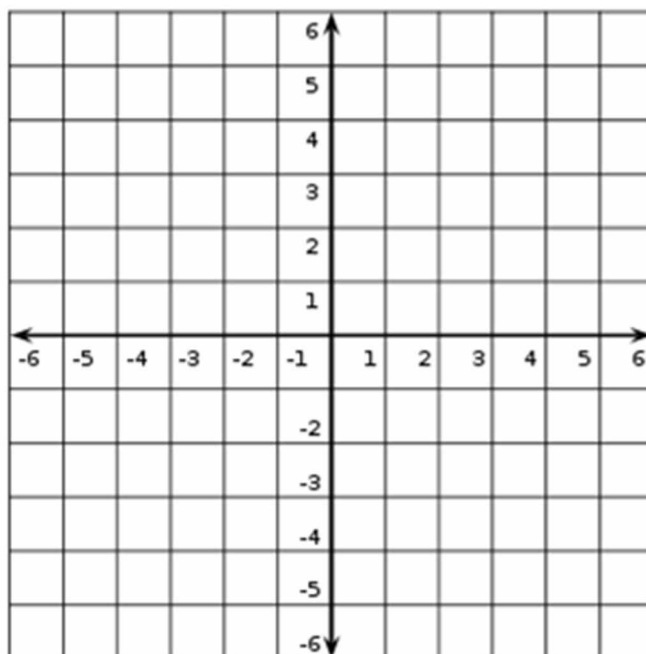
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(1) Solve each of the following systems graphically:

a) $x - 1 > 0$, $y \leq -2$



b) $x \geq 0$, $y - 2x < 3$

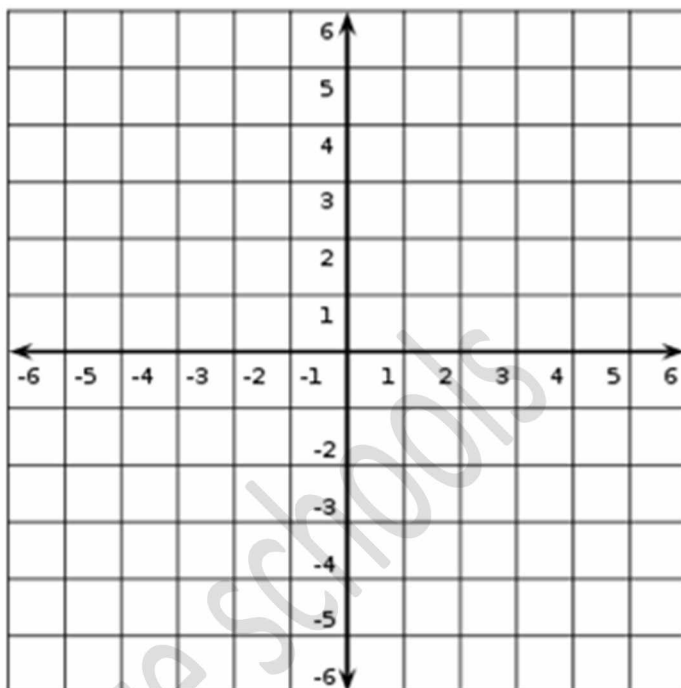


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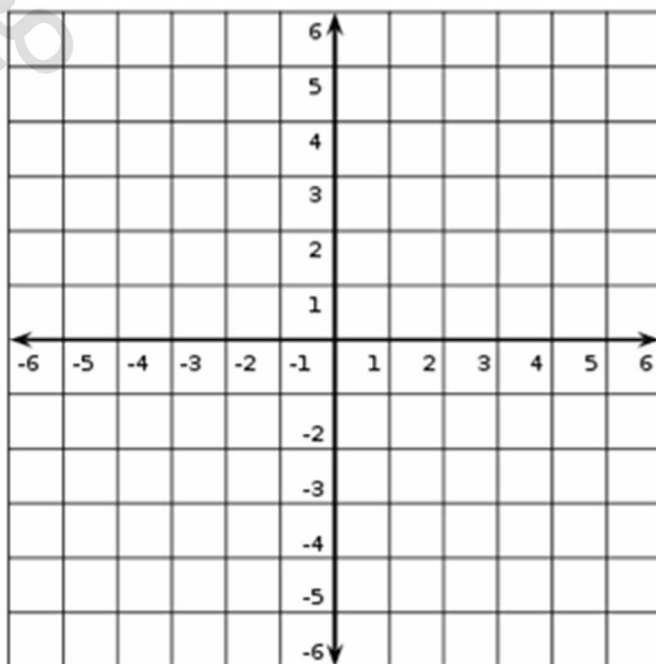


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c) $-2 < x \leq 1$, $1 \leq y < 5$



d) $x - 3y \geq 1$, $6y \geq 2 + 2x$



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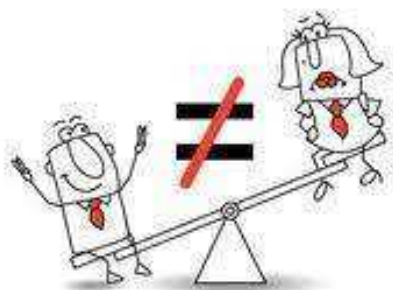
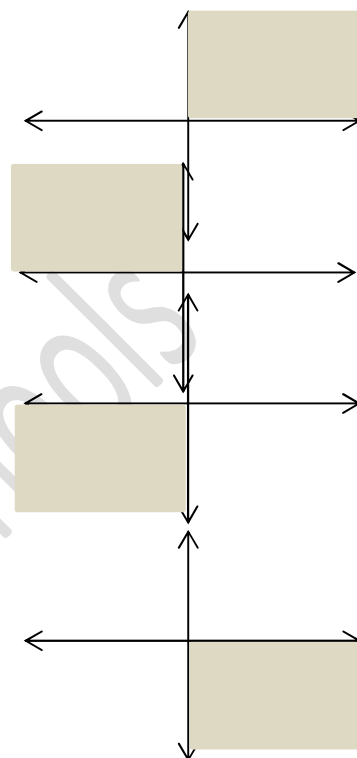
Remarks:

☐ $x \geq 0, y \geq 0$ represents the 1st quadrant

☐ $x \leq 0, y \geq 0$ represents the 2nd quadrant

☐ $x \leq 0, y \leq 0$ represents the 3rd quadrant

☐ $x \geq 0, y \leq 0$ represents the 4th quadrant

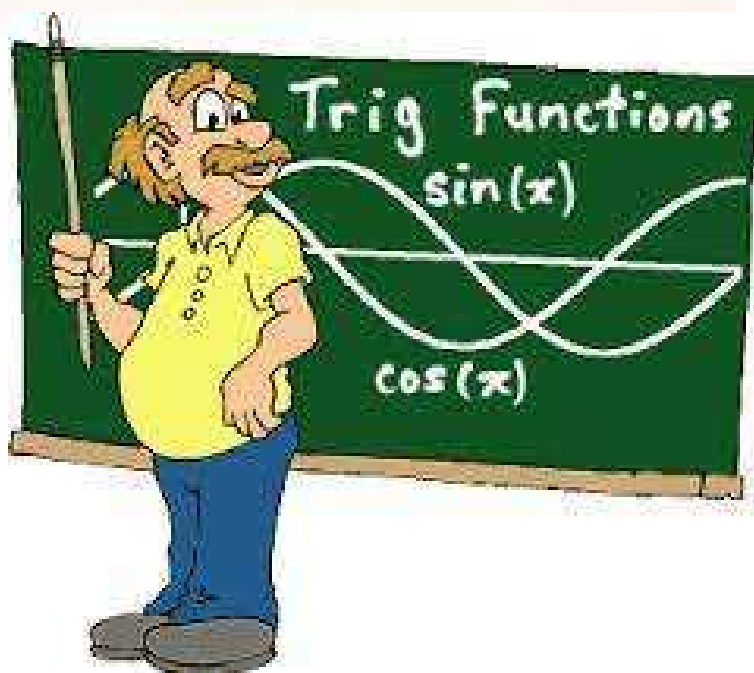


Inequality

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Lesson (1)**Trigonometric identities****➤ Trigonometric identity:**

It is an inequality which is true for all values of the variable

Ex) $\tan \theta = \frac{\sin \theta}{\cos \theta}$ is called identity because it is true for all values of θ

➤ Inequality: it is not true for all values of the variable

Ex) $\sin \theta = \frac{1}{2}$

Basic trigonometric identities

$$(1) \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$(2) \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$(3) \sin \theta = \frac{1}{\csc \theta}, \csc \theta = \frac{1}{\sin \theta} \text{ and } \cos \theta = \frac{1}{\sec \theta}, \sec \theta = \frac{1}{\cos \theta}$$

$$(4) \tan \theta = \frac{1}{\cot \theta}, \cot \theta = \frac{1}{\tan \theta}$$

(5) From the unit circle:

$$x^2 + y^2 = 1 \text{ then } \boxed{\sin^2 \theta + \cos^2 \theta = 1}$$

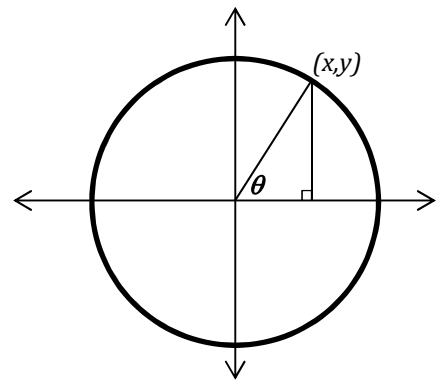
Dividing by $\cos^2 \theta$

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} \text{ then } \tan^2 \theta + 1 = \sec^2 \theta$$

$$\boxed{\sec^2 \theta = 1 + \tan^2 \theta}$$

Dividing by $\sin^2 \theta$

$$\boxed{\csc^2 \theta = 1 + \cot^2 \theta}$$



Sheet (1)**1** Which of the following relations represents an equation and which of them represents an identity :

(1) $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$

(2) $\cot \theta = \frac{-1}{\sqrt{3}}$

(3) $\tan\left(\frac{3\pi}{2} + \theta\right) = -\cot \theta$

(4) $\cos\left(\frac{3\pi}{2} - \theta\right) = -\sin \theta$

(5) $\sin^2 \theta + \cos^2 \theta = 1$

(6) $\sin(2\pi - \theta) = -\frac{1}{2}$

2 Choose the correct answer from the given ones :(1) $\cos(90^\circ - \theta) \sec(\theta - 90^\circ)$ in the simplest form equals

(a) 1

(b) -1

(c) $\sin^2 \theta$ (d) $\cot^2 \theta$ (2) The expression : $\frac{1 - \cos^2 \beta}{\sin^2 \beta - 1}$ in the simplest form equals(a) $-\tan^2 \beta$ (b) $-\cos^2 \beta$ (c) $\tan^2 \beta$ (d) $\cot^2 \beta$ (3) $\frac{1 + \tan^2 \theta}{1 + \cot^2 \theta}$ in the simplest form equals(a) $\tan^2 \theta$ (b) $\cot^2 \theta$

(c) 1

(d) $\cos^2 \theta$ (4) $\sin^2 \theta + \cos^2 \theta + \tan^2 \theta =$

(a) 1

(b) $\cot^2 \theta$ (c) $\csc^2 \theta$ (d) $\sec^2 \theta$ (5) $(\tan^2 \theta - \sec^2 \theta)^5 =$

(a) 1

(b) -1

(c) 5

(d) -5

(6) $2 \sin^2 \theta + \cos^2 \theta + \frac{1}{\sec^2 \theta} =$

(a) 2

(b) 1

(c) $\tan^2 \theta$ (d) $\sec^2 \theta$ (7) $\sin \theta \csc \theta + 2 \cos \theta \sec \theta + 3 \tan \theta \cot \theta =$

(a) 1

(b) 3

(c) 5

(d) 6

(8) In ΔABC , if $\sin^2 A + \cos^2 B = 1$, then ΔABC is

(a) equilateral.

(b) isosceles.

(c) scalene.

(d) right-angled.

3 Complete the following “where θ is the measure of an angle in which all trigonometric functions and their reciprocals are defined at it” :

(1) $\sin \theta \csc \theta = \dots\dots\dots$

(2) $\cos \theta = \frac{1}{\dots\dots\dots}$

(3) $\cot \theta \tan \theta = \dots\dots\dots$

(4) $\frac{\sin \theta}{\cos \theta} = \dots\dots\dots$

(5) $\sin^2 \theta + \cos^2 \theta = \dots\dots\dots$

(6) $\sin^2 \theta = 1 - \dots\dots\dots$

(7) $\tan^2 \theta + 1 = \dots\dots\dots$

(8) $\cot^2 \theta + 1 = \dots\dots\dots$

4 Write in the simplest form each of the following expressions “where θ is the measure of an angle in which all trigonometric functions and their reciprocals are defined at it” :

(1) $\sin \theta + \cos \theta)^2 - 2 \sin \theta \cos \theta$

(2) $\sin \left(\frac{\pi}{2} + \theta \right) \sec (-\theta)$

(3) $\cos^2 \theta \sec \theta \csc \theta$

(4) $\sin \theta \csc \theta - \cos^2 \theta$

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5 Prove the validity of each of the following identities :

(1) $\sin (90^\circ - \mu) \cos \mu = 1 - \sin^2 \mu$

(2) $\cot^2 \mu - \cos^2 \mu = \cot^2 \mu \cos \mu^2$

(3) $\sec^2 \beta + \csc^2 \beta = \sec^2 \beta \csc^2 \beta$

(4) $\sec \theta - \sin \theta \tan \theta = \cos \theta$

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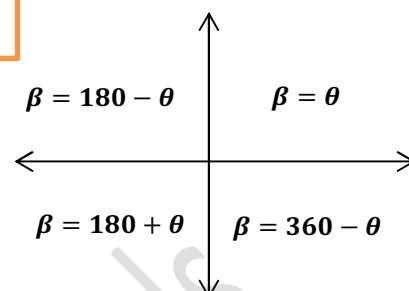
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Lesson (2)Solving trigonometric equations**First: finding the general solution****Steps:**

- Determine the quadrant
- Find the angle " shift"
- Add $2n\pi$



- The general solution the equation : $\cos \theta = a$ is $\theta = \pm \beta + 2\pi n$
- The general solution the equation : $\sin \theta = a$ is
 $\theta = \beta + 2\pi n$, $\theta = (\pi - \beta) + 2\pi n$
- The general solution the equation : $\tan \theta = a$ is $\theta = \beta + \pi n$

Sheet (2)**1 Complete the following :**

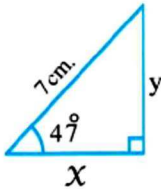
- The general solution of the equation : $\sin \theta = 1$ for all values of θ is
- The general solution of the equation : $\cos \theta = 1$ for all values of θ is
- The general solution of the equation : $\sin \theta = -1$ for all values of θ is
- The general solution of the equation : $\cos \theta = -1$ for all values of θ is
- The general solution of the equation : $\sin \theta = 0$ for all values of θ is
- The general solution of the equation : $\cos \theta = 0$ for all values of θ is
- The general solution of the equation : $\tan \theta = 1$ for all values of θ is
-  The general solution of the equation : $\sin \theta = \cos \theta$ for all values of θ is
- The solution set of the equation : $\sin \theta = \frac{1}{2}$, where $\theta \in]0, \frac{\pi}{2}[$ is

2 Choose the correct answer :

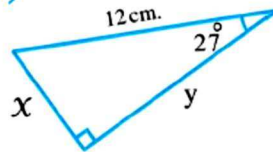
- (1) If $0^\circ \leq \theta < 360^\circ$ and $\sin \theta + 1 = 0$, then $\theta = \dots\dots\dots$
 (a) 0° (b) 90° (c) 180° (d) 270°
- (2) If $0^\circ \leq \theta < 360^\circ$ and $\cos \theta + 1 = 0$, then $\theta = \dots\dots\dots$
 (a) 90° (b) 180° (c) 270° (d) 360°
- (3) If $0^\circ \leq \theta < 360^\circ$ and $\csc \theta - 1 = 0$, then $\theta = \dots\dots\dots$
 (a) 0° (b) 90° (c) 180° (d) 270°
- (4) The solution set of the equation : $\sqrt{3} \tan \theta = 1$, where $90^\circ < \theta < 270^\circ$ is $\dots\dots\dots$
 (a) $\{30^\circ\}$ (b) $\{150^\circ\}$ (c) $\{210^\circ\}$ (d) $\{240^\circ\}$
- (5) The solution set of the equation : $\sin \theta + \cos \theta = 0$, where $180^\circ < \theta < 360^\circ$ is $\dots\dots\dots$
 (a) $\{210^\circ\}$ (b) $\{225^\circ\}$ (c) $\{240^\circ\}$ (d) $\{315^\circ\}$
- (6) If $\theta \in [0, \pi[$, $\cot \theta = 1$, then $\theta = \dots\dots\dots$
 (a) 30° (b) 45° (c) 60° (d) 135°
- (7) If $\theta \in [0, \frac{\pi}{2}[$, $\sin \theta \cot \theta = \frac{1}{2}$, then the solution set is $\dots\dots\dots$
 (a) $\emptyset$ (b) $\{\frac{\pi}{3}\}$ (c) $\{\frac{4\pi}{3}\}$ (d) $\{\frac{5\pi}{3}\}$
- (8) The solution set of the equation : $\sin^2 \theta + 1 = 0$, $\theta \in [0, \pi[$, is $\dots\dots\dots$
 (a) $\{90^\circ\}$ (b) $\{0^\circ\}$ (c) $\{180^\circ\}$ (d) $\emptyset$
- (9) If $180^\circ \leq \theta < 360^\circ$ and $2 \cos \theta + 1 = 0$, then $\theta = \dots\dots\dots$
 (a) 210° (b) 240° (c) 300° (d) 330°
- (10) The general solution of the equation : $\tan \theta = \frac{1}{\sqrt{3}}$ is $\dots\dots\dots$ (where $n \in \mathbb{Z}$)
 (a) $\frac{\pi}{6} + n\pi$ (b) $2n\pi \pm \frac{\pi}{6}$ (c) $\frac{\pi}{3} + n\pi$ (d) $2n\pi \pm \frac{\pi}{3}$
- (11) The general solution of the equation : $\cos \theta = \frac{1}{2}$ is $\dots\dots\dots$ (where $n \in \mathbb{Z}$)
 (a) $2n\pi \pm \frac{\pi}{3}$ (b) $2n\pi \pm \frac{\pi}{6}$ (c) $\frac{\pi}{6} + n\pi$ (d) $\frac{\pi}{3} + n\pi$

Lesson (3)**Solving the right-angled triangle****1** Find the value of each of x and y in each of the following figures :

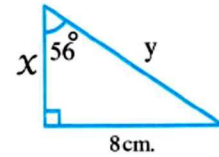
(1)



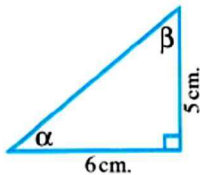
(2)



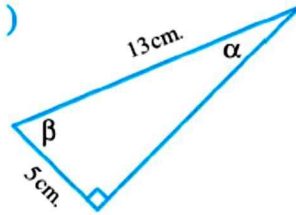
(3)

**2** Find the value of each of the angles α and β in degree measure in each of the following figures :

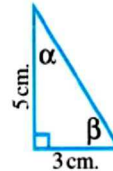
(1)



(2)



(3)

**3** ABC is a right-angled triangle at B. Find AB to one decimal , if :(1) $m(\angle C) = 32^\circ 18'$ and $AC = 25$ cm.

Sheet (3)**1** ABC is a right-angled triangle at B. Find AB to one decimal , if :

1 $m(\angle C) = 54^\circ 13'$ and $BC = 20$ cm.

« 27.7 cm. »

.....

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.....

2 ABC is a right-angled triangle at B. Find $m(\angle C)$ to the nearest minute , if :

1 $BC = 54$ cm. and $AC = 88$ cm.

« $52^\circ 9'$ »

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3 Solve the triangle ABC which is right-angled at B approximating the measures of angles to the nearest degree and the lengths of sides to the nearest cm. where :

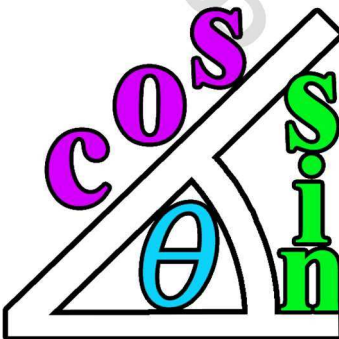
(1) $AB = 4$ cm , $BC = 6$ cm.

(2) $AB = 12.5$ cm , $BC = 17.6$ cm.

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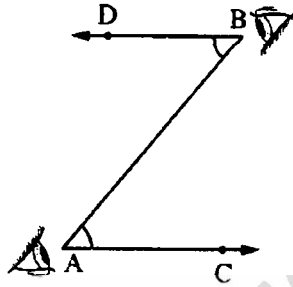
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Lesson (4)**Angles of elevation and angles of depression****Angle of elevation**

If a person looked from the point A to an object at the point B above his horizontal sight , then the included angle between the horizontal ray $\overrightarrow{AC}$ and the seeing ray to above $\overrightarrow{AB}$ is called the elevation angle of B with respect to A

i.e. $\angle CAB$ is the elevation angle of B with respect to A

**Angle of depression**

If a person looked from the point B to an object at the point A down his horizontal sight , then the included angle between the horizontal ray $\overrightarrow{BD}$ and the seeing ray to down $\overrightarrow{BA}$ is called the depression angle of A with respect to B

i.e. $\angle DBA$ is the depression angle of A with respect to B

Sheet (4)

- 1  From a point 8 metres apart from the base of a tree , it was found that the measure of the elevation angle of the top of the tree is 22°

Find the height of the tree to the nearest hundredth.

« 3.23 m. »

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- 2 A man found that the measure of the angle of elevation of the top of a tower ,
at a distance of 50 m. from its base , is $39^{\circ} 21'$ Find the height of the tower. « 41 m. »

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- 3 The length of the thread of a kite is 42 metres. If the measure of the angle which the
thread makes with the horizontal ground equals 63° , find to the nearest metre the height
of the kite from the surface of the ground. « 37 m. »

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- 4 A person observed , from the top of a hill 2.56 km. high , a point on the ground. He
found its depression angle measure was 63° . Find the distance between the point and the
observer to the nearest metre. « 2873 m. »

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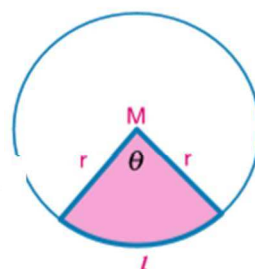
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Lesson (5)**The Circular sector**

The circular sector: is a part of the surface of the circle bounded by two radii and an arc .

Area of the circular sector = $\frac{1}{2} r^2 \theta^{\text{rad}}$ (where θ is the angle of the sector, r is the radius of the circle)

**Example**

- ① Find the area of the circular sector in which the length of the radius of its circle is 10cm and the measure of its angle is 1.2^{rad}

● **Solution**

Formula:

$$\text{Area of the circular sector} = \frac{1}{2} r^2 \theta^{\text{rad}}$$

Substituting $r = 10$, $\theta^{\text{rad}} = 1.2^{\text{rad}}$:

$$= \frac{1}{2} (10)^2 \times 1.2 = 60 \text{ cm}^2$$

Remember

Relation between the degree measure and the radian measure is:

$$\frac{\theta^{\text{rad}}}{\pi} = \frac{x^{\circ}}{180^{\circ}}$$

Example

- ② A circular sector in which the length of the radius of its circle equals 16cm, and the measure of its angle equals 120° , find its area to the nearest square centimetre .

● **Solution**

Formula:

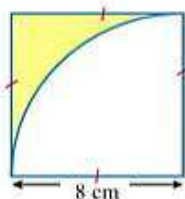
$$\text{area of the sector} = \frac{x^{\circ}}{360^{\circ}} \times \pi r^2$$

Substituting $r = 16$, $x^{\circ} = 120^{\circ}$:

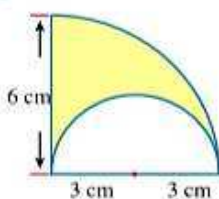
$$= \frac{120^{\circ}}{360^{\circ}} \times \pi (16)^2 \simeq 268 \text{ cm}^2$$

1 Find in terms of π the area of the shaded part in each of the following figures:

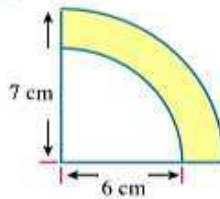
A



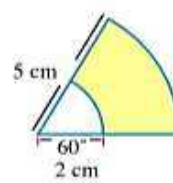
B



C



D



2 Find to the nearest cm^2 the area of a circular sector, where the measure of its central angle is 30° and the radius of its circle is of length 3.5 cm. « 3 cm^2 approximately »

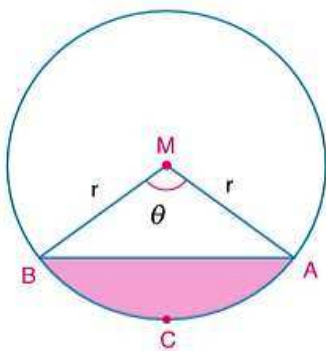
3 Find the area of the circular sector in which the length of the radius of its circle is 10 cm. and the measure of its angle is 1.2^{rad} « 60 cm^2 »

Sheet (5)**1 Choose the correct answer from the given ones :**

- (1) The area of the circular sector =
 (a) $\frac{1}{2} l r^2$ (b) $\frac{1}{2} r \theta^{\text{rad}}$
 (c) the area of the circle $\times \frac{\theta^{\text{rad}}}{2\pi}$ (d) the area of the circle $\times \frac{x^\circ}{180^\circ}$
- (2) The area of a sector whose arc is of length 10 cm. and the length of the diameter of its circle = 10 cm. equals
 (a) 50 cm² (b) 25 cm² (c) 12.5 cm² (d) 100 cm²
- (3) The area of the circular sector in which the measure of its angle is 1.2^{rad} and the length of the radius of its circle is 4 cm. equals
 (a) 4.8 cm² (b) 9.6 cm² (c) 12.8 cm² (d) 19.6 cm²
- (4) The perimeter of the circular sector in which the length of its arc is 4 cm. and the length of the diameter of its circle is 10 cm. equals
 (a) 14 cm. (b) 20 cm. (c) 30 cm. (d) 40 cm.
- (5) The area of the circular sector in which the measure of its angle is 120° , the length of the radius of its circle is 3 cm. equals
 (a) 3 π cm² (b) 6 π cm² (c) 9 π cm² (d) 12 π cm²
- (6) The area of the circular sector in which , its perimeter is 12 cm. , length of its arc is 6 cm. equals
 (a) 6 cm² (b) 9 cm² (c) 12 cm² (d) 18 cm²
- (7) If the perimeter of a sector is 8 cm. and its arc is of length 2 cm. , then its circle is of radius length
 (a) 6 cm. (b) 2 cm. (c) 3 cm. (d) 4 cm.
- (8) The arc of a sector is of length 3 cm. and the area of this sector is 15 cm² , then its circle radius is of length
 (a) 5 cm. (b) 10 cm. (c) 2.5 cm. (d) 15 cm.
- (9) The perimeter of a sector is 44 cm. Its circle is of radius length 14 cm. , then the length of the arc of the sector =
 (a) 16 cm. (b) 8 cm. (c) 32 cm. (d) 4 cm.
- (10) If the area of the circular sector equals 110 cm² , the measure of its angle equals 2.2^{rad} , then the length of the radius of its circle equals
 (a) 2 cm. (b) 5 cm. (c) 10 cm. (d) 20 cm.

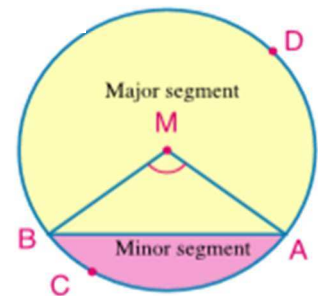
Lesson (6)**Circular Segment**

The **circular segment** is a part of the surface of the circle bounded by an arc and a chord passing by the ends of this arc.

Finding the area of the circular segment:

$$\text{Area of the circular segment} = \frac{1}{2} r^2 (\theta^{\text{rad}} - \sin \theta)$$

Where r is the length of the radius of its circle, θ is the measure of the angle of the segment.

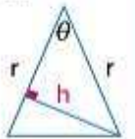
**Remember**

Area of the triangle

$$= \frac{1}{2} r \times h \text{ where:}$$

$$\sin \theta = \frac{h}{r}$$

$$h = r \sin \theta$$



Area of the triangle =

$$\frac{1}{2} \times r \times r \sin \theta$$

Example

- ① Find the area of the circular segment whose length of the radius of its circle equals 8cm, and the measure of its angle equals 150° .

Solution

$$\theta^{\text{rad}} = 150^\circ \times \frac{\pi}{180^\circ} \approx \frac{5\pi}{6}$$

$$\sin \theta = \sin 150^\circ$$

$$\text{Area of the circular segment} = \frac{1}{2} r^2 (\theta^{\text{rad}} - \sin \theta)$$

$$\text{Area of the circular segment} = \frac{1}{2} \times 64 \left(\frac{5\pi}{6} - \sin 150^\circ \right) \approx 67.7758 \text{ cm}^2$$

Sheet (6)**1 Complete :**

- (1) The circular segment is
- (2) The area of the circular segment =
- (3) The area of the circular segment whose radius length is 10 cm. and its arc is of length 5 cm. is
- (4) The area of the circular segment equals the area of the circular sector subtended by the same arc if its central angle is of measure
- (5) ABC is a triangle in which : $AB = 5$ cm. , $BC = 8$ cm. , $m(\angle B) = 60^\circ$, then the area of $\Delta ABC = \dots\dots\dots \text{cm}^2$

2 Find the area of the circular segment in which :

- (1) The length of its chord equals 6 cm. , and the length of the radius of its circle equals 5 cm. « 4 cm² approximately »
- (2) Its height equals 5 cm. , and the length of the radius of its circle equals 10 cm. « 61 cm² approximately »

- 3 A chord of length 6 cm. is drawn in a circle of radius length 6 cm. Find the area of the minor segment. « 3.26 cm² approximately »

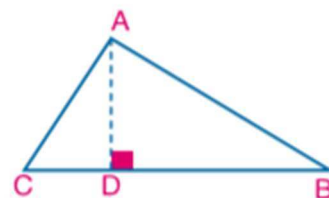
- 4 The area of a circle is 706.5 cm² Find the area of a segment of this circle where the measure of its angle is 135° « 185.52 cm² approximately »

Lesson (7) : Areas

The Area of a triangle in terms of the lengths of two sides and the included angle

From the area of the triangle:

$$\begin{aligned}\text{Area of the triangle} &= \frac{1}{2} BC \times AD \\ &= \frac{1}{2} \times BC \times AB \sin B\end{aligned}$$



In general:

Area of the triangle = half the product of the lengths of two sides $\times$ sine the included angle between them.

Example

- ① Find the area of the triangle ABC in which $AB = 9$ cm, $AC = 12$ cm, $m(\angle A) = 48^\circ$ approximating the result to the nearest hundredth.

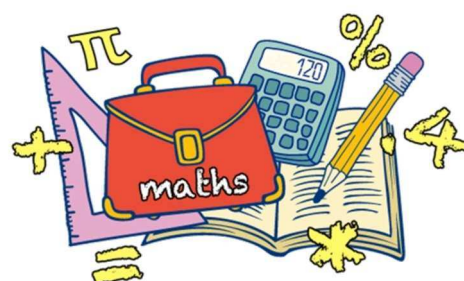
Solution

$$\text{Area of the triangle } ABC = \frac{1}{2} \times AB \times AC \sin A$$

$$\text{Substituting } AB = 9 \text{ cm, } AC = 12 \text{ cm, } m(\angle A) = 48^\circ$$

$$\text{Area of the triangle } ABC = \frac{1}{2} \times 9 \times 12 \times \sin 48 \simeq 40.13 \text{ cm}^2$$

→ 1 ÷ 2 × 9 × 12 × Sin 48 =



Sheet (7)

- 1 Find the area of the triangle ABC in which : $AB = 8 \text{ cm.}$, $AC = 10 \text{ cm.}$
and $m(\angle A) = 48^\circ$ approximating the result to the nearest hundredth.

« 29.73 cm^2 »

.....

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.....

- 2 The area of the equilateral triangle ABC is $36\sqrt{3} \text{ cm}^2$, then find its side length.

« 12 cm. »

.....

.....

.....

- 3 Find the area of the quadrilateral in which the lengths of its diagonals are 12 cm. ,
16 cm. and the measure of the included angle between them is 68° approximating the
result to the nearest square centimetre.

« 89 cm^2 »

.....

.....

- 4 Find the area of each of the following regular polygons approximating the result to
the nearest tenth :

(1) A regular pentagon of side length equals 16 cm.

« 440.4 cm^2 »

(2) A regular hexagon of side length equals 12 cm.

« 374.1 cm^2 »

.....

.....

.....

.....

Unit Summary

The identity: is true equality for all real values of the variable which each of the two sides of the equality is known.

Pythagorean identities: $\sin^2 \theta + \cos^2 \theta = 1$, $1 + \tan^2 \theta = \sec^2 \theta$, $1 + \cot^2 \theta = \csc^2 \theta$

Prove the validity of the identity: to prove the validity of trigonometric identity, we prove that the two functions determining its two sides are equal.

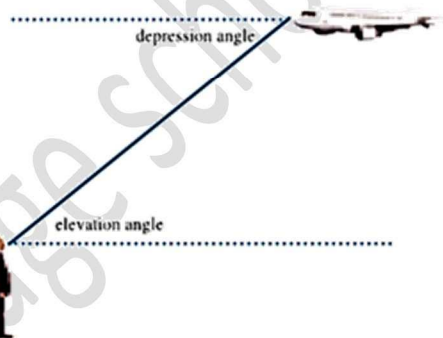
The function: is a true equality for some real numbers which satisfies this equality and is not true for some other which is not satisfy it.

Elevation angle and depression angle:

Elevation or depression angle is the union of the horizontal ray and the initial ray from the body passing through the eye of the observer.

Measure of the elevation angle = measure of the depression angle.

(alternate).



The circular sector: is a part of the surface of the circle bounded by the two radii and an arc.

Area of the circular sector

$$\begin{aligned}
 &= \frac{1}{2} r^2 \theta^{\text{rad}} \quad (\text{where } \theta^{\text{rad}} \text{ is the angle of the sector, } r \text{ is the radius of its circle}) \\
 &= \frac{x^\circ}{360^\circ} \times \text{Area of the circle} \quad (\text{where } x^\circ \text{ is the degree measure of the angle of the sector}) \\
 &= \frac{1}{2} l r \quad (\text{where } l \text{ is the length of the arc, } r \text{ is the radius of its circle})
 \end{aligned}$$

The circular segment : is a part of the surface of the circle bounded by an arc in it and a chord passes through its ends of this arc.

Area of the segment $= \frac{1}{2} r^2 (\theta^{\text{rad}} - \sin \theta)$

(where θ is the measure of the central angle of the segment, r is the radius of its circle).

Area of the triangle $= \frac{1}{2} \text{ length of the base} \times \text{height}$

$$= \frac{1}{2} \text{ Product of its sides} \times \sin \text{ the included angle between them.}$$

Area of the quadrilateral $= \frac{1}{2} \text{ product of its diagonals} \times \sin \text{ the included angle between them.}$

Area of the regular polygon $= \frac{1}{4} n x^2 \times \cot \frac{\pi}{n}$

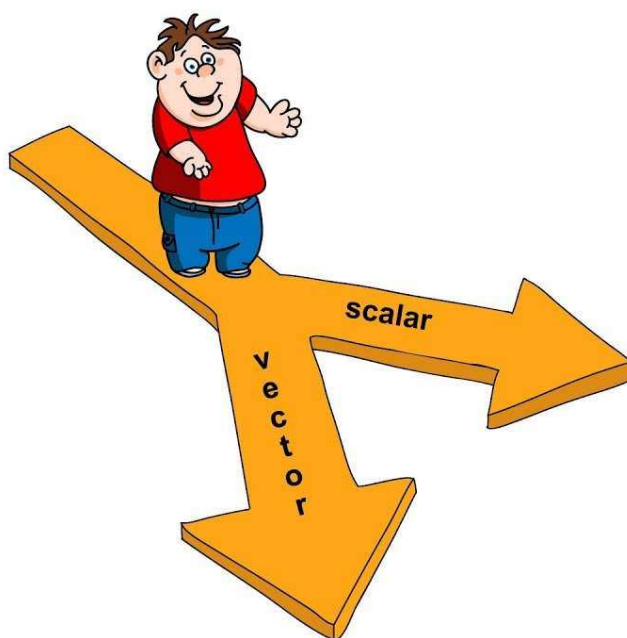
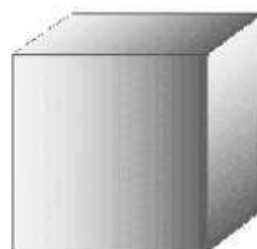
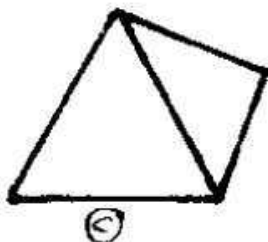
(where n is the number of its sides, x is the length of its side)

Date ://



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Analytic Geometry

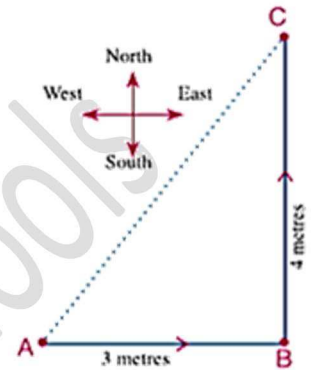


Lesson (1)**Scalars, Vectors & Directed line segment****Scalar quantities**

Scalar quantities are determined completely by their magnitude only such as length, area ...

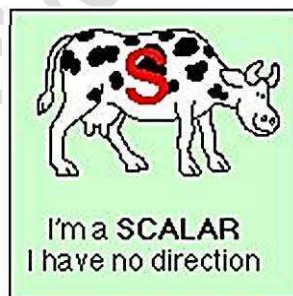
Vector quantities

- Vector quantities are determined completely by their magnitude and
- their direction such as velocity, force. ...

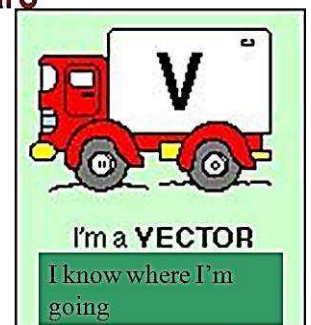
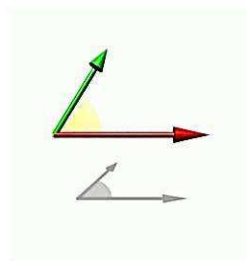
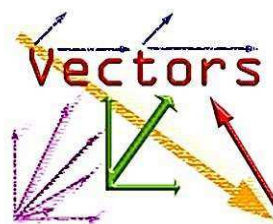
**Notice that:**

- **Distance** is a scalar quantity which is the result of $AB + BC$ or $CB + BA$.
- **Displacement** is the distance between the starting and ending points only and in direction from A to C. i.e to describe the displacement, its magnitude AC and its direction from A to C must be determined.

Displacement is a vector quantity which is the distance covered in a certain direction.

**Vector Addition**

$$R = A + B$$

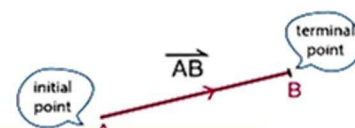
**Vectors and Scalars**



Definition 1 The directed line segment: is a line segment which has an initial point, an terminal point and a direction.



Definition 2 The norm of the directed line segment: norm of $\overrightarrow{AB}$ is the length of $\overline{AB}$ and is denoted by the symbol $\|\overrightarrow{AB}\|$.



Notice that: $\|\overrightarrow{AB}\| = \|\overrightarrow{BA}\| = AB$



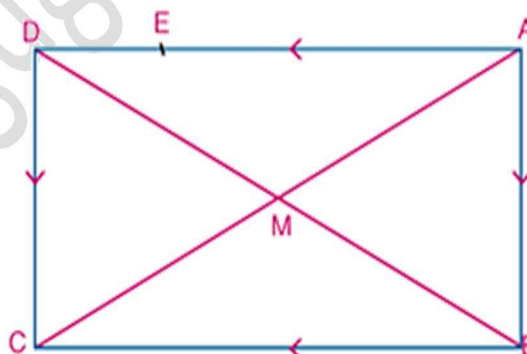
Definition 3 Equivalent directed line segments: Two directed line segments are said to be equivalent if they have the same norm and same direction.

Example

- 1 In the figure opposite: ABCD is a rectangle, its diagonals are intersecting at M. $E \in \overline{AD}$ then:

$\overline{AB} \parallel \overline{CD}$, $AB = CD$, $\overline{BC} \parallel \overline{AD}$, $BC = AD$ and

$MA = MC = MB = MD$



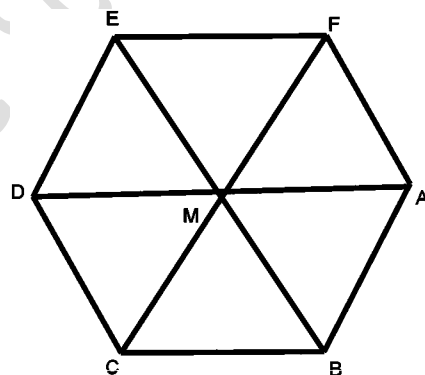
- A $\because \|\overrightarrow{AB}\| = \|\overrightarrow{DC}\|$, $\overrightarrow{AB}$ and $\overrightarrow{DC}$ have the same direction
 $\therefore \overrightarrow{AB}$ is equivalent to $\overrightarrow{DC}$
- B $\because \|\overrightarrow{AM}\| = \|\overrightarrow{MC}\|$, $\overrightarrow{AM}$ and $\overrightarrow{MC}$ have the same direction
 $\therefore \overrightarrow{AM}$ is equivalent to $\overrightarrow{MC}$
- C $\because \|\overrightarrow{MA}\| = \|\overrightarrow{MB}\|$, $\overrightarrow{MA}$ and $\overrightarrow{MB}$ have different direction
 $\therefore \overrightarrow{MA}$ is not equivalent to $\overrightarrow{MB}$
- D $\because \|\overrightarrow{AE}\| \neq \|\overrightarrow{CB}\|$, $\overrightarrow{AE}$ and $\overrightarrow{CB}$ have the same direction
 $\therefore \overrightarrow{AE}$ is not equivalent to $\overrightarrow{CB}$

Sheet (1)**1 Complete :**

- 1 to define scalar quantity you should know
- 2 to define vector quantity you should know
- 3 the directed line segment is a line segment which has,,
- 4 two directed line segment are equivalent if they have
- 5 in the opposite figure :

ABCDEF is a regular hexagon , then

- a) $\overrightarrow{AB}$ is equivalent to And equivalent to
- b) $\overrightarrow{MD}$ is equivalent to And equivalent to
- c) $\overrightarrow{MD}$ is equivalent to And equivalent to

**2 On the lattice** , if : A(3,-2) , B(6,2) , C(1,3) , D(4,7)

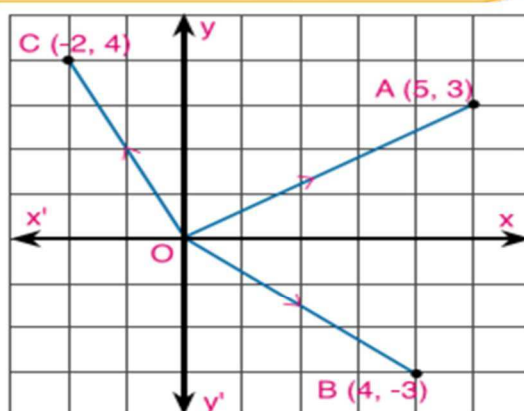
- a) Find : $\|\overrightarrow{AB}\|$ and $\|\overrightarrow{CD}\|$
- b) prove that : $\overrightarrow{AB}$ equivalent to $\overrightarrow{CD}$

Lesson (2)**Vectors****Position Vector**

Definition The position vector of a given point with respect to the origin point is the directed line segment which its starting point is the origin point and the given point is its terminal point.

A (5, 3), B(4, -3), C (-2, 4) then:

- $\vec{OA}$ is the position vector of the point A with respect to the origin point O, and corresponding to the ordered pair (5, 3) and is written as $\vec{OA} = (5, 3)$.

**Norm of the vector:**

Is the length of the line segment representing to the vector.

If: $\vec{R} = (x, y)$

Then: $\|\vec{R}\| = \sqrt{x^2 + y^2}$

Polar form of position Vector

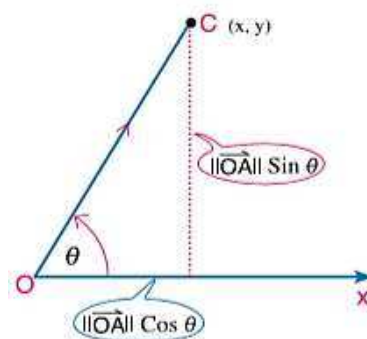
In the figure opposite: the vector $\vec{OA}$ makes θ with the positive direction of the x-axis and its norm equals $\|\vec{OA}\|$. It is possible to express it as follows:

$$\vec{OA} = (\|\vec{OA}\|, \theta)$$

Polar form of the vector.

the coordinates of point A in the orthogonal coordinate plane are:

$$x = \|\vec{OA}\| \cos \theta, \quad y = \|\vec{OA}\| \sin \theta, \quad \tan \theta = \frac{y}{x}$$



The unit vector : it is a vector whose norm is unity.

Zero vector : it is a vector whose norm equals zero and denoted by $\vec{0} = (0, 0)$

Parallel and perpendicular vector

For every non zero vectors $\vec{A} = (x_1, y_1)$ and $\vec{B} = (x_2, y_2)$

1) if $\vec{A} \parallel \vec{B}$

Then $\tan \theta_1 = \tan \theta_2$

And $\frac{y_1}{x_1} = \frac{y_2}{x_2}$

And $x_1 y_2 - x_2 y_1 = 0$

2) if $\vec{A} \perp \vec{B}$

Then $\tan \theta_1 \times \tan \theta_2 = -1$

And $\frac{y_1}{x_1} \times \frac{y_2}{x_2} = -1$

And $x_1 x_2 + y_1 y_2 = 0$

Example

If $\vec{A} = (6, -8)$, $\vec{B} = (-9, 12)$ and $\vec{C} = (-4, -3)$

(1) Prove that : $\vec{A} \parallel \vec{B}$, $\vec{B} \perp \vec{C}$, $\vec{C} \perp \vec{A}$

.....
.....
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Example

If $\vec{M} = (3, 2)$ and $\vec{N} = (2, k)$, **find the value of k in each of the two cases :**

(1) $\vec{M} \parallel \vec{N}$

(2) $\vec{M} \perp \vec{N}$

.....
.....
.....



Sheet (2)**1 Complete the following :**

- (1) The position vector of a given point is
- (2) The fundamental unit vector $\hat{i}$ is the directed line segment to the origin point and its norm is and its direction is
- (3) If $\vec{A} = (4, 5)$ and $\vec{B} = (3, -2)$, then $2\vec{A} + \vec{B} = \dots\dots\dots$
- (4) If $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} - \hat{j}$, then $2\vec{A} - \vec{B} = \dots\dots\dots$
- (5) If $\vec{E} = \vec{O}$ and $\vec{E} = (2 - a, b + 3)$, then $a = \dots\dots\dots$, $b = \dots\dots\dots$
- (6) If $\vec{A} = (5, -12)$, then $\|\vec{A}\| = \dots\dots\dots$

Choose the correct answer from the given ones :

- (1) If $\vec{A} + \vec{B} = (8, 16)$ and $\vec{A} = (5, 12)$, then $\|\vec{B}\| = \dots\dots\dots$
- (a) 7 (b) 5 (c) 13 (d) $8\sqrt{5}$
- (2) All the following vectors are unit vectors except
- (a) $(1, 0)$ (b) $(0.6, 0.8)$ (c) $(0, -1)$ (d) $(1, 1)$
- (3) If $\|k(3, 4)\| = 1$, then $k = \dots\dots\dots$
- (a) $\frac{1}{7}$ (b) $\frac{1}{25}$ (c) $\pm \frac{1}{5}$ (d) ± 5
- (4) The vector $\vec{M} = (8\sqrt{2}, \frac{\pi}{4})$ is expressed in terms of the fundamental unit vectors by the form
- (a) $4\hat{i} + 4\hat{j}$ (b) $8\hat{i} - 8\hat{j}$ (c) $-4\hat{i} - 8\hat{j}$ (d) $8\hat{i} + 8\hat{j}$
- (5) If $\vec{A} = (4, 5)$ and $\vec{B} = (-20, 16)$, then the two vectors $\vec{A}$ and $\vec{B}$ are
- (a) perpendicular. (b) parallel. (c) equivalent. (d) otherwise.
- (6) If $\vec{L} = (2, -3)$ and $\vec{K} = (3, 1 - x)$ are parallel, then $x = \dots\dots\dots$
- (a) 4 (b) $\frac{11}{2}$ (c) -1 (d) -9
- (7) If $\vec{A} = (x, 4)$, $\vec{B} = (2, y)$ and $\vec{A} \parallel \vec{B}$, then
- (a) $x + 2y = 0$ (b) $x = 2y$ (c) $xy = 8$ (d) $\frac{-}{y} \cdot 2$

If $\vec{A} = (6, -8)$, $\vec{B} = (-9, 12)$ and $\vec{C} = (-4, -3)$

(1) Prove that : $\vec{A} \parallel \vec{B}$, $\vec{B} \perp \vec{C}$, $\vec{C} \perp \vec{A}$

(2) Find : $2\vec{A} + \vec{B}$, $\vec{B} - 2\vec{C}$, $\frac{1}{2}\vec{A} + \vec{B} - 3\vec{C}$

.....
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If $\vec{M} = (3, 2)$ and $\vec{N} = (2, k)$, **find the value of k in each of the two cases :**

(1) $\vec{M} \parallel \vec{N}$

(2) $\vec{M} \perp \vec{N}$

.....
.....
.....

If $\| -8\vec{A} \| = 5 \| k\vec{A} \|$, **find the value of : k**

.....
.....

Find the polar form of each of the following vectors :

(1) $\vec{M} = 8\sqrt{3}\vec{i} + 8\vec{j}$

(2) $\vec{N} = 3\sqrt{2}\vec{i} + 3\sqrt{2}\vec{j}$

(3) $\vec{OA} = (5, 5\sqrt{3})$

(4) $\vec{B} = (7\sqrt{3}, -7)$

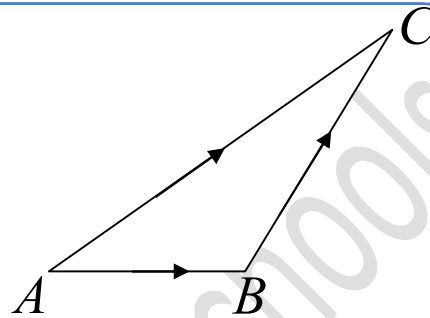
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Lesson (3)**Operation On Vectors****First**

Adding vectors geometrically

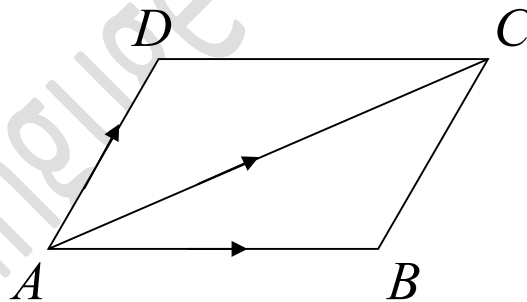
1] the triangle rule :

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$



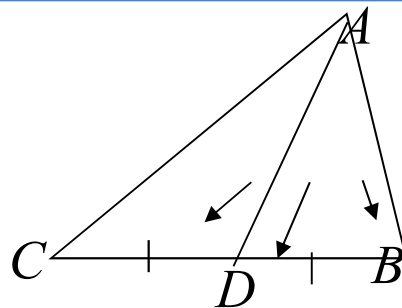
2] the parallelogram rule :

$$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$$



3] the median rule:

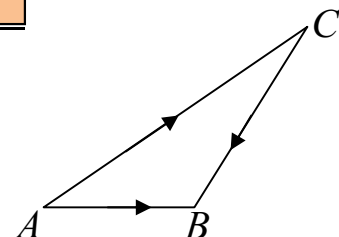
$$\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$$

**Second**

Subtracting two vectors geometrically

$$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$$

$$\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A}$$



Example**In the quadrilateral ABCD , prove that :**

$$(1) \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD} \quad | \quad (2) \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{DC} + \overrightarrow{AD}$$

.....

.....

.....



Sheet (3)**1 Complete :**

1 if: $\vec{A} = (-1, 5)$, $\vec{B} = (2, 1)$, then $\|\vec{AB}\| = \dots\dots$

2 if: $\vec{A} = (4, -2)$, $\vec{AB} = (3, 5)$, then $\vec{B} = \dots\dots$

3 if: M is a midpoint of $\overline{XY}$, then $\overline{XM} + \overline{YM} = \dots\dots$

4 if: ABC is a triangle, then $\overline{AB} + \overline{BC} + \overline{CA} = \dots\dots$

5 if: ABC is a triangle, then $\overline{AB} - \overline{CB} = \dots\dots$, $\overline{BA} - \overline{BC} = \dots\dots$

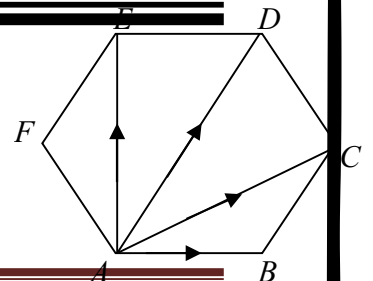
2 ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$, E is the midpoint of $\overline{AB}$ F is the midpoint of $\overline{DC}$.prove that : $\overline{AD} + \overline{BC} = 2 \overline{EF}$ 3 ABCD is a quadrilateral in which : $\overline{BC} = 3 \overline{AD}$.prove that :

a) ABCD is a trapezium

b) $\overline{AC} + \overline{BD} = 4 \overline{EF}$

4 ABCDEF is regular hexagon prove that :

$$\overline{AB} + \overline{AC} + \overline{AE} + \overline{AF} = 2 \overline{AD}$$



Lesson (4)**Application on Vectors****First Geometric applications**

We know that if $\overrightarrow{AB} = k \overrightarrow{DC}$, $k \neq 0$, then $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are :

- carried by the same straight line

I.e. : A , B , C , D are collinear.

or

- carried by two parallel straight lines

I.e. : $\overrightarrow{AB} // \overrightarrow{DC}$

Remark

If ABCD is a quadrilateral in which $\overrightarrow{AB} = k \overrightarrow{DC}$, $k \neq 0$, then

$\overrightarrow{AB} // \overrightarrow{DC}$, $\| \overrightarrow{AB} \| = |k| \| \overrightarrow{DC} \|$ and vise versa.

Example

Use vectors to prove that : the points A (1, 4), B(-1, -2), C(2, -3) are vertices of right angled triangle at B.

.....

Example

Use the vectors to prove that: the points A (3, 4), B(1, -1), C(-4, -3), D(2, 2) are vertices of a rhombus.

.....

Second Physical applications**1 The resultant force**

- **The force :** is a vector passes through a given point and acts along a straight line.
- **The force :** is represented by a directed line segment and it is drawn by a suitable drawing scale.

For example :

- 1** A force of magnitude $F_1 = 10$ Newton acts in the East direction.

$$\vec{F}_1 = 10 \vec{e}$$

$\vec{F}_1$ is represented by a directed line segment of length 2 cm.

Remember that :

- 1** Consider $\vec{e}$ a unit vector in the East direction.
- 2** Choose a suitable drawing scale "Each 5 Newton is represented on drawing by 1 cm".

**Example**

If the forces: $\vec{F}_1 = 2\vec{i} + \vec{j}$, $\vec{F}_2 = \vec{i} + 7\vec{j}$, $\vec{F}_3 = \vec{i} - 5\vec{j}$ act on a particle, Calculate the magnitude and direction of their resultant (forces are measured in Newton).

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.....

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

Relative Velocity**Example**

A car (A) moves on a straight road with speed 70 km/h, A car (B) moves on the same road with speed 90 km/h. Find the relative velocity of car (A) with respect to car (B) when:

- A** The two cars move in the same direction.
B The two cars move in the opposite direction.

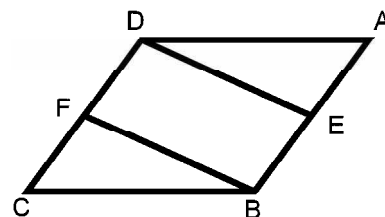
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Sheet (4)**First****Geometry**

1 ABCD is a parallelogram ,E is a midpoint of AB

F is a midpoint of DC

Prove that : DEBF is a parallelogram

2 ABCD is a quadrilateral , if $\overrightarrow{AC} + \overrightarrow{BD} = 2 \overrightarrow{DC}$ prove that :

ABCD is a parallelogram

3 using vectors prove that : A(3,4) , B(1,-1) , C(-4,-3) , D(-2,2)

are vertices of a rhombus

4 using vectors prove that : A(1,3) , B(6, 1) , C(4,-4) , D(-1,-2)

are vertices of a square and find its area.

5 ABCD is a trapezium , AD//BC

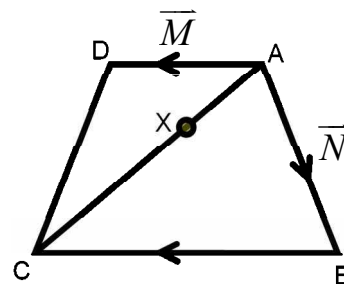
$$AD = \frac{1}{2} BC, \overrightarrow{AB} = \vec{N}, \overrightarrow{AD} = \vec{M}$$

a) Express in term of $\vec{M}$ and $\vec{N}$ each of the following :

$$\overrightarrow{BC}, \overrightarrow{AC}, \overrightarrow{DC}, \overrightarrow{DB}$$

b)if : $X \in \overline{AC}$ where $AX = \frac{1}{3} \times AC$

prove that : the point D , X and B are collinear .



Second Physical application**1** Complete:

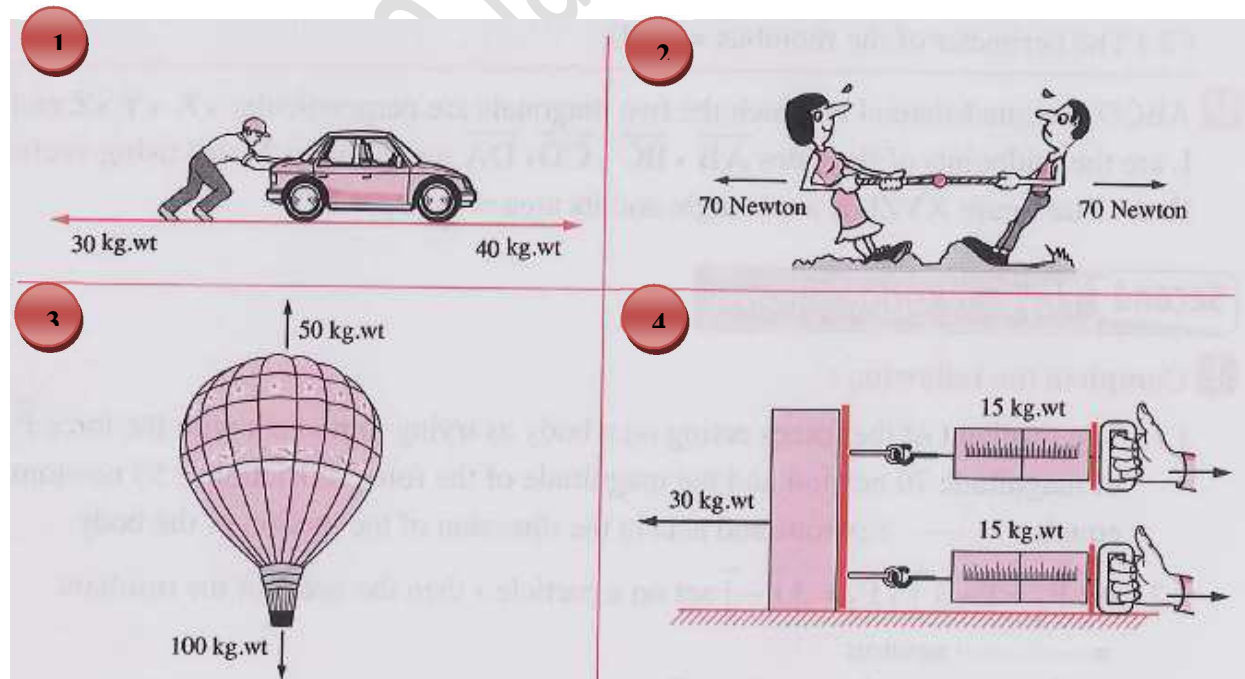
1 If: $\vec{F}_1 = i - 3j$, $\vec{F}_2 = 3i - j$ act on a particle, then the norm of the resultant =N

2 If: $\vec{F}_1 = (a, b)$, $\vec{F}_2 = -3i + 4j$ act on a particle and the system is in equilibrium, then $a = \dots\dots$, $b = \dots\dots$

3 If: $\vec{V}_A = 12\vec{e}$, $\vec{V}_B = 8\vec{e}$, then $\vec{V}_{AB} = \dots\dots$

4 If: $\vec{V}_A = 120\vec{e}$, $\vec{V}_B = -80\vec{e}$, then $\vec{V}_{BA} = \dots\dots$, $\vec{V}_{AB} = \dots\dots$

5 If: $\vec{V}_{AB} = 75\vec{e}$, $\vec{V}_A = -60\vec{e}$, then $\vec{V}_{BA} = \dots\dots$, $\vec{V}_B = \dots\dots$

2 Find the resultant force $\vec{F}$ acting in each of the following:

3 In each of the following, the two forces $\vec{F}_1$ and $\vec{F}_2$ act at a particle. Show the magnitude and the direction of the resultant of each two forces:

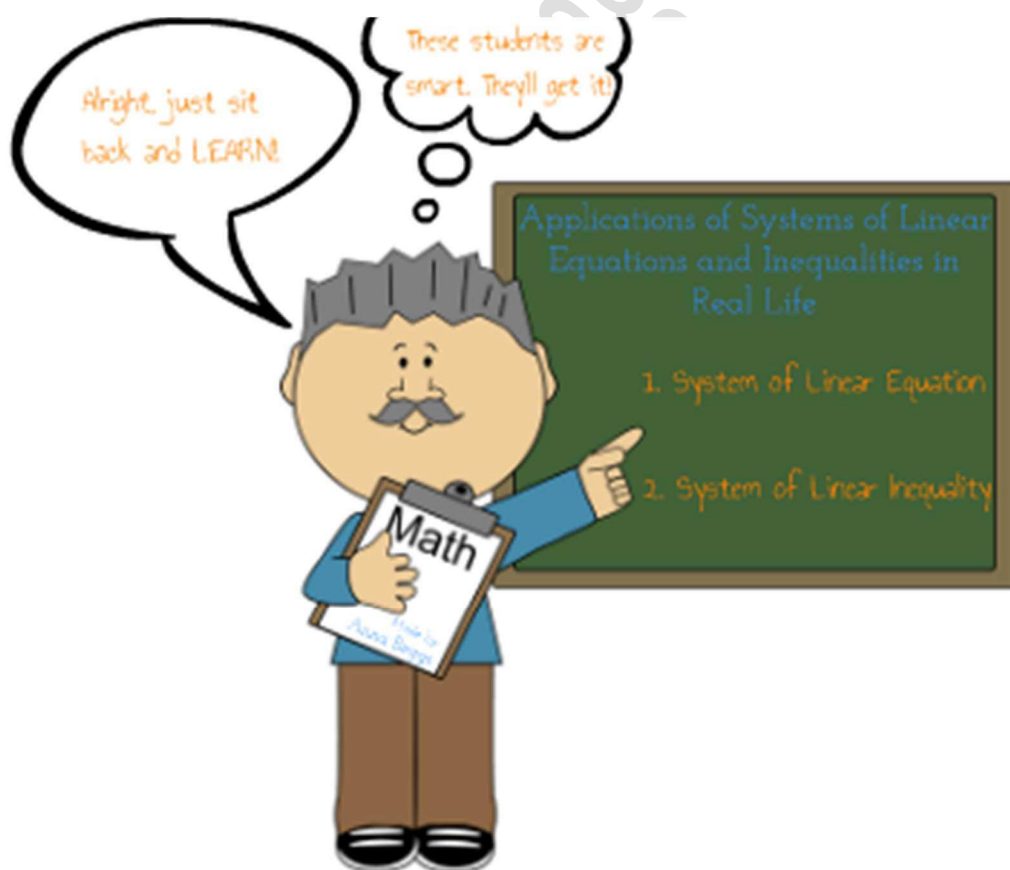
- 1 $F_1 = 15$ newtons acts in the east direction,
 $F_2 = 40$ newtons acts in the west direction.
- 2 $F_1 = 34$ gm.wt. acts in the north east direction,
 $F_2 = 34$ gm.wt. acts in the south west direction.
- 3 $F_1 = 50$ dyne acts in 60° west of the north direction,
 $F_2 = 50$ dyne acts in 30° south of the east direction.
- 4 $F_1 = 30$ newtons acts in 20° east of the north direction ,
 $F_2 = 30$ newtons acts in 70° north of the east direction.

4 Forces $\vec{F}_1 = 7\mathbf{i} - 5\mathbf{j}$, $\vec{F}_2 = a\mathbf{i} + 3\mathbf{j}$, $\vec{F}_3 = -4\mathbf{i} + (b-3)\mathbf{j}$, find the values of a and b if:

- (1) The system of forces are in equilibrium.
- (2) The resultant of the forces = $-5\mathbf{j}$



ANALYTICAL GEOMETRY



Lesson (1)Division of a line segment

First: Finding the Coordinates of the point of division of a line segment by a certain ratio:

1- Internal division

If $C \in \overline{AB}$, then point C

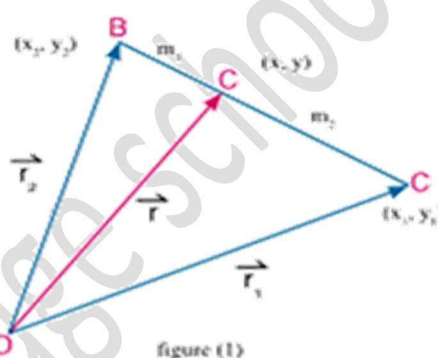
divides $\overline{AB}$ internally by the ratio $m_2 : m_1$

where $\frac{m_2}{m_1} > 0$ then $\frac{AC}{CB} = \frac{m_2}{m_1}$

and for the two directed segments $\overrightarrow{AC}$, $\overrightarrow{CB}$

The same direction i.e.: $m_1 \times \overrightarrow{AC} = m_2 \times \overrightarrow{CB}$

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x, y)$



Then

$$\overrightarrow{r} (m_1 + m_2) = m_1 \overrightarrow{r_1} + m_2 \overrightarrow{r_2}$$

i.e.:

$$\overrightarrow{r} = \frac{m_1 \overrightarrow{r_1} + m_2 \overrightarrow{r_2}}{m_1 + m_2}$$

which is called the
vector form

Example

- ① If $A(2, -1)$, $B(-3, 4)$, find the coordinates of point C which divides $\overline{AB}$ internally by the ratio $3 : 2$ in the vector form.

Solution

Let $C(x, y)$

$$\therefore A(2, -1) \quad \therefore \overrightarrow{r_1} = (2, -1) \quad , \quad \therefore B(-3, 4) \quad \therefore \overrightarrow{r_2} = (-3, 4)$$

$$m_2 : m_1 = 3 : 2$$

$$\therefore \overrightarrow{r} = \frac{m_1 \overrightarrow{r_1} + m_2 \overrightarrow{r_2}}{m_1 + m_2}$$

$$\therefore \overrightarrow{r} = \frac{2(2, -1) + 3(-3, 4)}{2 + 3} = \frac{(4, -2) + (-9, 12)}{5} = \frac{(-5, 10)}{5} = (-1, 2)$$

$\therefore$ The coordinates of point C are $(-1, 2)$

Cartesian form:

$$(x, y) = \frac{m_1(x_1, y_1) + m_2(x_2, y_2)}{m_1 + m_2} = \frac{(m_1 x_1 + m_2 x_2, m_1 y_1 + m_2 y_2)}{m_1 + m_2}$$

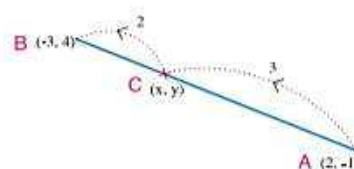
From that we get: $(x, y) = \left(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}, \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} \right)$

Example

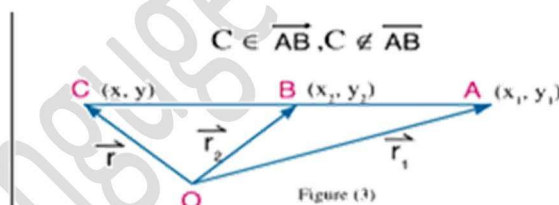
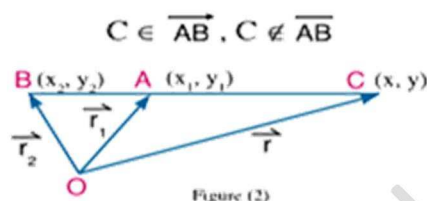
- ② Solve the previous example using the Cartesian form.

Solution

$$(x, y) = \left(\frac{2 \times 2 + 3 \times -3}{2 + 3}, \frac{2 \times -1 + 3 \times 4}{2 + 3} \right) = (-1, 2)$$

**2- External division**

If $C \in \overrightarrow{AB}$, $C \notin \overline{AB}$, then C divides $\overrightarrow{AB}$ externally by the ratio $m_2 : m_1$ where $\frac{m_2}{m_1} < 0$ then one of the two values m_1 or m_2 is positive and the other is negative, then the following figure illustrates that there are two probabilities:

**Example**

- ③ If A (2, 0), B (1, -1), find the coordinates of point C which divides $\overrightarrow{AB}$ externally by the ratio 5 : 4.

Solution

$$\therefore \vec{r}_1 = (2, 0), \vec{r}_2 = (1, -1)$$

$$m_2 : m_1 = 5 : -4 \therefore \frac{m_2}{m_1} < 0 \text{ negative}$$

$$\vec{r} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

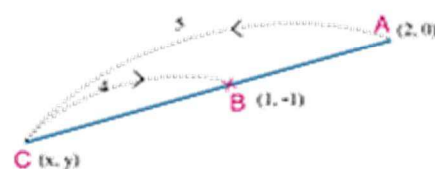
$$\therefore \vec{r} = \frac{-4(2, 0) + 5(1, -1)}{-4 + 5}$$

$$\vec{r} = (-8 + 5, 0 - 5) = (-3, -5)$$

$\therefore$ The coordinates of point C are (-3, -5)

Cartesian form:

$$(x, y) = \left(\frac{-4 \times 2 + 5 \times 1}{-4 + 5}, \frac{-4 \times 0 + 5 \times -1}{-4 + 5} \right) = (-3, -5)$$

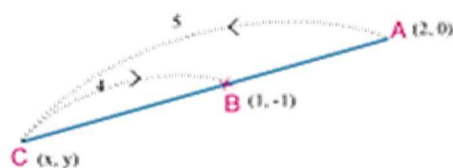


by substituting $C(x, y)$

mathematical formula for the rule

by distributing

by adding and simplifying



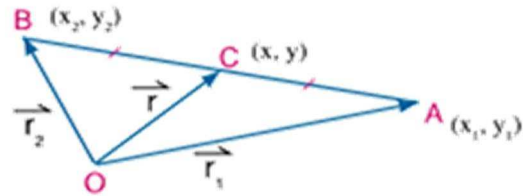
Notice that:

If C is the midpoint of $\overline{AB}$ where A (x_1, y_1) , B (x_2, y_2)
then: $m_1 = m_2 = m$ then

$$\vec{r} = \frac{\vec{r}_1 + \vec{r}_2}{2}$$

Vector form

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Cartesian form**Second : Finding the ratio of Division**

If point C divides $\overline{AB}$ by the ratio $m_2 : m_1$ and:

- 1- The ratio of division $\frac{m_2}{m_1} > 0$ then the division is internal.
- 2- The ratio of division $\frac{m_2}{m_1} < 0$ then the division is external.

Example

- ④ If A $(5, 2)$, B $(2, -1)$, find the ratio by which $\overline{AB}$ is divided by the points of intersection of $\overline{AB}$ with the two axes, showing the type of division in each case, then find the coordinates of the division point.

Solution

First: let the x-axis intersects $\overline{AB}$ at point C $(x, 0)$

where $\frac{AC}{CB} = \frac{m_2}{m_1}$ then: $y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

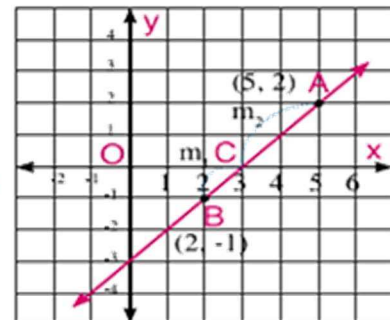
$$\therefore 0 = \frac{m_1(2) + m_2(-1)}{m_1 + m_2} = \frac{2m_1 - m_2}{m_1 + m_2}$$

$$\therefore 2m_1 = m_2 \quad \therefore \frac{m_2}{m_1} = \frac{2}{1} \quad (\text{ratio of division})$$

$$\therefore \frac{m_2}{m_1} > 0$$

$\therefore$ The division is internal by the ratio 2 : 1

$$\therefore \text{The coordinates are } C \left(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}, 0 \right) = \left(\frac{1 \times 5 + 2 \times 2}{1 + 2}, 0 \right) = (3, 0)$$



Second: The straight line intersects the y-axis at point D

Let the coordinates of D be $(0, y)$

where $\frac{AD}{DB} = \frac{m_2}{m_1}$ then $x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

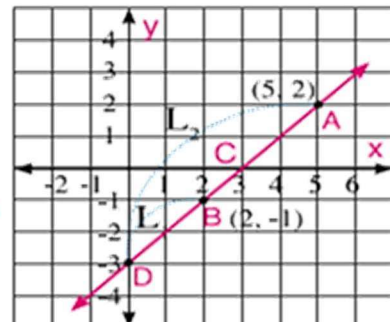
$$\therefore 0 = \frac{m_1 \times 5 + m_2 \times 2}{m_1 + m_2}$$

$$\therefore 2m_2 = -5m_1 \quad \therefore \frac{m_2}{m_1} = -\frac{5}{2} \quad (\text{ratio of division})$$

$$\therefore \frac{m_2}{m_1} < 0$$

$\therefore$ The division is external by the ratio 5 : 2

$$\text{The coordinates of the point D are } (0, y) = \left(0, \frac{-2 \times 2 + 5 \times -1}{-2 + 5} \right) = (0, -3)$$

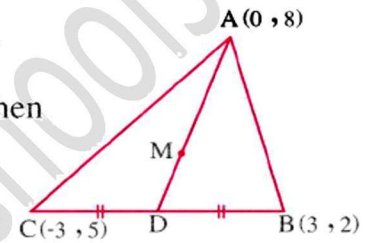


Sheet (1)**1 Complete the following :**

- (1) If $A = (3, 6)$, $B = (-7, 4)$, then the midpoint of $\overline{AB} = (\dots\dots\dots, \dots\dots\dots)$
- (2) If M is the point of intersection of the two diagonals of the parallelogram $ABCD$ where $A = (3, 7)$, $C = (-3, 1)$, then $M = (\dots\dots\dots, \dots\dots\dots)$
- (3) If the point $(3, 6)$ is the midpoint of $\overline{AB}$ where $A = (-3, 7)$, then the point $B = (\dots\dots\dots, \dots\dots\dots)$

(4) In the opposite figure :

$\overline{AD}$ is a median in $\triangle ABC$, M is the point of intersection of its medians where $A = (0, 8)$, $B = (3, 2)$, $C = (-3, 5)$, then the point $D = (\dots\dots\dots, \dots\dots\dots)$
the point $M = (\dots\dots\dots, \dots\dots\dots)$



- 2** If $A = (-3, -7)$, $B = (4, 0)$, find the coordinates of the point C which divides $\overline{AB}$ by the ratio $5 : 2$ internally.

« (2, -2) »

.....

.....

- 3**  If $A = (0, -3)$, $B = (3, 6)$, find the coordinates of the point C which divides $\overline{BA}$ internally by the ratio $1 : 2$

« (2, 3) »

.....

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- 4** If $A = (4, 3)$, $B = (-3, 5)$, find the point $C \in \overline{AB}$ where $3 AC = 5 CB$

.....

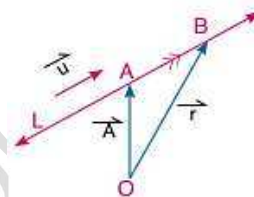
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Lesson (2)**Equation of straight line****Equation of the straight line given a point belonging to it and a direction vector to it****First: Vector form**

$$\vec{r} = \vec{A} + K \vec{u}$$

**Example**

- 1 Write the vector equation of the straight line which passes through point (2, -3) and its direction vector is (1, 2).

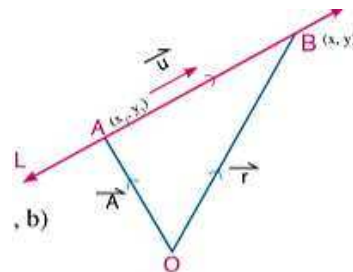
SolutionLet the straight line pass through point A (2, -3) and $\vec{u} = (1, 2)$

$$\therefore \vec{r} = \vec{A} + K \vec{u}$$

vector form of the equation of the straight line.

 $\therefore$ The vector equation of the straight line is $\vec{r} = (2, -3) + K(1, 2)$.**Second: The parametric equations**The vector equation is $\vec{r} = \vec{A} + K \vec{u}$

$$x = x_1 + k a \quad , \quad y = y_1 + k b$$

**Third : Cartesian Equation**Eliminating K from the parametric equations : $x = x_1 + ka$, $y = y_1 + kb$ We get the equation: $\frac{x - x_1}{a} = \frac{y - y_1}{b}$ i.e.: $\frac{b}{a} = \frac{y - y_1}{x - x_1}$ Put $\frac{b}{a} = m$ (where m is the slope of the line), then the equation becomes in the form: $m = \frac{y - y_1}{x - x_1}$ **Example**

- 3 Find the Cartesian equation of the straight line which passes through the point (3, -4) and its direction vector is (2, -1)

Solution

$$m = \frac{-1}{2}$$

$$\text{Slope of the line } m = \frac{b}{a}$$

$$m = \frac{y - y_1}{x - x_1}$$

equation of the line given its slope and a point belonging to it.

$$\frac{-1}{2} = \frac{y - (-4)}{x - 3}$$

$$m = \frac{-1}{2}, x_1 = 3, y_1 = -4$$

$$2y + 8 = -x + 3$$

Product of means = product of extremes:
general form.

$$x + 2y + 5 = 0$$

Date ://



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Sheet (2)

Find the equation of the S.t line

- 1 Passing through (1 , 3) and its slope = $-\frac{2}{3}$

.....
.....

- 2 Passing through the point (3 , -2) and its slope is -2

.....
.....
.....

- 3 Passing through the two points (3 , 1) and (5 , 4)

.....
.....
.....

- 4 Passing through the point (0 , -5) and makes with the positive direction of X - axis an angle of measure 135° .

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.....

- 5 Passing through the point (-2 , 1) and parallel to the straight line

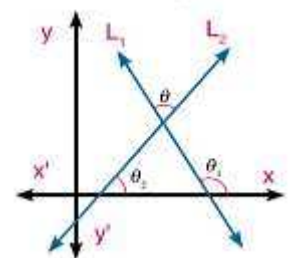
$$\vec{r} = (2, -3) + k(1, 0)$$

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Lesson (3)**The angle between two**

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \text{ where } m_1 m_2 \neq -1$$

- ① Find the measure of the acute angle between the two straight lines whose equations are
 $3x - 4y - 11 = 0$, $x + 7y + 5 = 0$

**Solution**

A We find the slope of each straight line:

$$m_1 = \frac{-3}{-4} = \frac{3}{4}$$

slope of the first line

$$m_2 = \frac{-1}{7}$$

slope of the second line

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Formula

$$\tan \theta = \left| \frac{\frac{3}{4} - (-\frac{1}{7})}{1 + \frac{3}{4}(-\frac{1}{7})} \right|$$

substituting the values of m_1, m_2

$$= \left| \frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{28}} \right| = \left| \frac{\frac{21+4}{28}}{\frac{28-3}{28}} \right| = 1$$

$$\theta = 45^\circ$$

Remember
 Slope of the straight line whose equation $ax + by + c = 0$ equals $-\frac{a}{b}$



Sheet (3)**1 Find the measure of the acute angle between the two straight lines whose slopes are :**

(1) $\frac{-3}{4}, -7$

(2) $\frac{1}{2}, \frac{2}{9}$

(3) $\frac{3}{4}, -\frac{2}{3}$

.....

.....

.....

2 Find the measure of the acute angle between each of the following pairs of straight lines :

(1) $L_1 : \vec{r} = (0, -2) + k(3, -1)$, $L_2 : \vec{r} = (0, 5) + k(2, 1)$

(2) $L_1 : \vec{r} = k(1, 0)$, $L_2 : \vec{r} = (3, -2) + k(1, -2)$

(3) $L_1 : \vec{r} = (0, 1) + k(1, 1)$, $L_2 : 2x - y - 3 = 0$

(4) $L_1 : 2x + 3y = 15$, $L_2 : \vec{r} = (-2, -1) + k(1, -3)$

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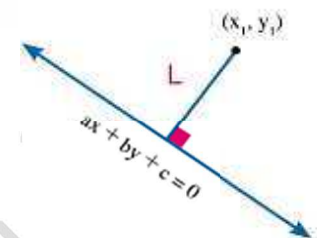
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Lesson (4)The length of the perpendicular from a point to a line

Finding the length of the perpendicular from a point to a straight line

$$L = \frac{|a x_1 + b y_1 + c|}{\sqrt{a^2 + b^2}}$$

**Example**

- 1 Find the length of the perpendicular from the point (4, -5) to the straight line $\vec{r} = (0, 2) + K(4, 3)$.

Solution

Let $(x, y) = (0, 2) + K(4, 3)$

$\therefore x = 4K, y = 2 + 3K$ (parametric equations to the vector equation)

$$\frac{x}{4} = \frac{y-2}{3}$$

by eliminating K

$$3x = 4y - 8$$

Product of means = product of extremes

$$3x - 4y + 8 = 0$$

Cartesian equation

$$L = \frac{|a x_1 + b y_1 + c|}{\sqrt{a^2 + b^2}}$$

Formula

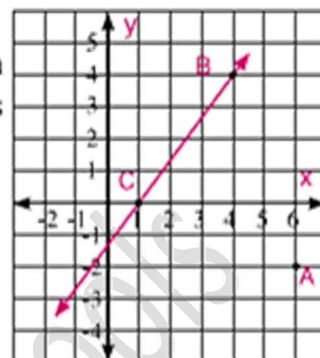
Substituting: $a = 3, b = -4, c = 8, x_1 = 4, y_1 = -5$

$$L = \frac{|3 \times 4 - 4 \times -5 + 8|}{\sqrt{3^2 + 4^2}}$$

$$= \frac{|12 + 20 + 8|}{\sqrt{9 + 16}} = \frac{|40|}{\sqrt{25}} = \frac{40}{5} = 8 \text{ unit of length}$$

Example

- ② In the figure opposite: Find the length of the perpendicular drawn from the point A (6, -2) to the straight line passing through the points B (4, 4), C (1, 0), then find the area of the triangle ABC.

**Solution**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Formula

$$\therefore C(1, 0), B(4, 4)$$

$$\therefore m = \frac{4 - 0}{4 - 1} = \frac{4}{3}$$

$$m = \frac{y - y_1}{x - x_1}$$

$$\frac{4}{3} = \frac{y - 0}{x - 1}$$

Substituting the point (4, 4), (1, 0)

equation of the line given the slope and a point belonging to it

substituting $m = \frac{4}{3}$

$$\text{Then: } 4x - 3y - 4 = 0$$

Cartesian equation

$$L = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

formula

length of the perpendicular from the point A (6, -2) to the line : $4x - 3y - 4 = 0$

$$\text{is: } L = \frac{|4 \times 6 - 3 \times -2 - 4|}{\sqrt{4^2 + 3^2}} = \frac{|24 + 6 - 4|}{\sqrt{25}} = \frac{26}{5} = 5 \frac{1}{5} \text{ unit of length}$$

Consider BC is the base of the triangle ABC

$$\therefore BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

formula

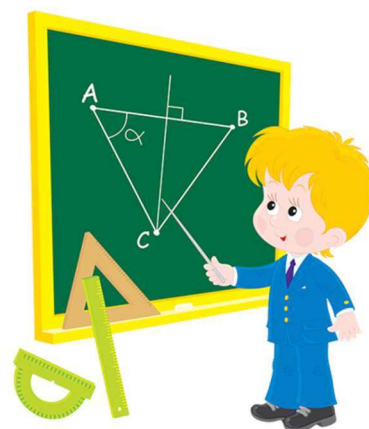
$$= \sqrt{(4 - 1)^2 + (4 - 0)^2} = 5 \text{ units}$$

substituting the points (4, 4), (1, 0)

$$\text{Area of the triangle ABC} = \frac{1}{2} \text{ length of base} \times \text{height}$$

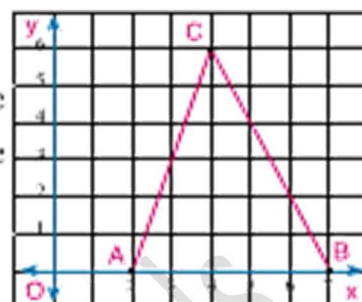
formula

$$= \frac{1}{2} \times 5 \times \frac{26}{5} = 13 \text{ square unit}$$



Sheet (4)**First : Complete each of the following:**

- 1 The figure opposite shows karim's house A (2, 0) and the school B (7, 0) and the mosque C (4, 6): Complete each of the following:
- A The equation of $\overrightarrow{AB}$ is
- B The length of $\overrightarrow{AB}$ equals
- C Shortest distance between the Mosque C and the road from the house to the school equals
- D Measure of the acute angle between the straight lines $\overrightarrow{AC}$ and $Y = 0$ equals
- E Area of $(\triangle ABC)$ equals

**Second : Multiple choice**

- 2 Length of perpendicular from the point $(-3, 5)$ on the y-axis equals
- A 2 B 3 C 5 D 8
- 3 The distance between the straight lines whose equations $y - 3 = 0$, $y + 2 = 0$ equals
- A 1 B 2 C 3 D 5
- 4 Length of perpendicular from the point $(1, 1)$ to the straight line whose equation $x + y = 0$ equals
- A 1 B $\sqrt{2}$ C 2 D $2\sqrt{2}$
- 5 If the length of perpendicular drawn from $(3, 1)$ to the straight line whose equation $3x - 4y + c = 0$ equals 2 unit of length, then C equals
- A Zero B 3 C 5 D 7
- 6 Find the length of the perpendicular drawn from A to the straight line L in exercises
- A - D
- A $A(0, 0)$, $L: \vec{r} = (0, 5) + t(3, 4)$
- B $A(2, -4)$, $L: 12x + 5y - 43 = 0$
- C $A(5, 2)$, $L: 8x + 15y - 19 = 0$
- D $A(-2, -1)$, $L: \vec{r} = (0, -7) + t(1, 2)$

Lesson (5)General equation of st.line passing through the point of the intersection of two lines**General equation of the straight line passing through the point of intersection of two given lines**

$$a_1 x + b_1 y + c_1 + k (a_2 x + b_2 y + c_2) = 0$$

Example

- ① Find the equation of the straight line passing through the point A (-2, 4) and the point of intersection of the two lines:

$$x + 2y - 5 = 0, \quad 2x - 3y + 4 = 0$$

Solution

$$a_1 x + b_1 y + c_1 + k (a_2 x + b_2 y + c_2) = 0$$

$$x + 2y - 5 + k (2x - 3y + 4) = 0$$

$$-2 + 2 \times 4 - 5 + k (2 \times -2 - 3 \times 4 + 4) = 0$$

$$1 - 12k = 0 \quad \text{i.e.} \quad k = \frac{1}{12}$$

$$x + 2y - 5 + \frac{1}{12} (2x - 3y + 4) = 0$$

$$12x + 24y - 60 + 2x - 3y + 4 = 0$$

$$14x + 21y - 56 = 0$$

$$2x + 3y - 8 = 0$$

general equation

substituting the two equations

substituting $x = -2, y = 4$

Simplify

Substituting the value of k

multiply both sides by 12

Simplify

Divide both sides by 7

Sheet (5)

- ① Find the vector equation of the straight line which passes through the origin point and the two straight lines whose equation $x = 3$, $y = 4$

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- ② Find the vector equation of the straight line which passes through the point $(3, 1)$, and the point of intersection of the two lines whose equations $3x + 2y - 7 = 0$, $x + 3y = 7$

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- ③ Find the equation of the straight line passes through the point of intersection of the two straight lines whose equations $\vec{r} = k(-3, 2)$, $3x - 2y = 13$ and parallel to the y-axis.....

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- ④ Find the equation of the straight line passes through the point of intersection of the two lines whose equations $2x + y = 5$, $x + 5y = 16$ and perpendicular to the line whose equation $x - y = 8$

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Notes

[illegible]

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امسح الباركود بالمويل لتثبيت
التطبيق وتحميل كل الملازم مجاناً وبسرعة



حمل التطبيق الآن



كورسات ونتائج امتحانات



ملازم ثانوي



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